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An Initial Study of the Application of the Numerical Method of Characteristics to Unsteady Flow Analysis in Partially Filled Gravity Drainage Sized Pipes

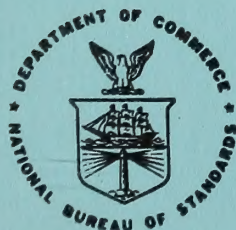
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**AN INITIAL STUDY OF THE APPLICATION
OF THE NUMERICAL METHOD OF
CHARACTERISTICS TO UNSTEADY FLOW
ANALYSIS IN PARTIALLY FILLED GRAVITY
DRAINAGE SIZED PIPES**

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U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, Secretary
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Director

Summary

The application of the numerical method of characteristics to the solution of the differential equations defining unsteady flow in partially filled drainage system sized pipes is outlined.

The derivation of the flow equations is presented, together with the necessary boundary equation formulation to represent variable inflow, system discharge and leakage flow past a stationary deposited solid.

A computer program, written in Fortran, is included, together with typical output, that establishes the applicability of this computational method to unsteady flow analysis in gravity flow drainage systems.

Proposals for the extension of the described techniques to the prediction of solid transport and flow attenuation in long pipes are also presented.

Preface

This report is one of a group documenting National Bureau of Standards (NBS) research and analysis efforts in developing water conservation test methods, analysis, economics, and strategies for implementation and acceptance. This work is sponsored by the Department of Housing and Urban Development/Office of Policy Development and Research, Division of Energy Building Technology and Standards, under HUD Interagency Agreement H-48-78.

Report prepared by Dr. J. A. Swaffield, Senior Lecturer, Drainage Research Group, Department of Building Technology, Brunel University, Uxbridge, UK., during a study leave period as a guest research worker at NBS/Stevens Institute of Technology.

Experimental results included in this report are drawn from the published work of the Drainage Research Group at Brunel University.

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Notation

A	Pipe flow cross section area
C^+, C^-	Notation referring to the positive and negative characteristics
c	Wave speed
Fr	Froude No $= V/\sqrt{gh}$
g	Acceleration due to gravity
h	Flow depth
K	Coefficient in solid leakage characteristics, $Q = K(SE - SE_0)^2$
m	Hydraulic mean depth
N, n	No of pipe length sections employed
P	Wetted channel perimeter
Q	Flow rate
SE	Specific energy $= h + V^2/2g$
SE_0	Minimum specific energy required to initiate flow past solid
S_0	Pipe slope
S	Slope of energy grade line, defined by Manning's Equation
T	Surface width of flow within partially filled channel
t	Time
V	Local mean velocity
X1-4	Functions of h, V, c and S calculated at each base point at each time step
x	Distance, + ve in initial flow direction
α	Pipe slope, $S_0 = \sin \alpha$
Δt	Time step
Δx	Pipe section length
θ	$\Delta t/\Delta x$

- λ Multiplier in combination equation yielding total differential unsteady flow relationship = $\pm \sqrt{\frac{g}{\Delta t}}$
- ρ Fluid density
- T_o Wall to fluid shear stress
- Suffixes
- A, B, C Calculated points in an x-t grid at time t
- P Calculated points in an x-t grid at time t + Δt
- R, S Interpolated points in an x-t grid at time t
- U, D Upstream and downstream conditions relative to the solid boundary

1. INTRODUCTION

Unsteady flow is defined as fluid flow wherein the flow parameters vary with time and as such will occur in all fluid carrying conduits if the system boundary conditions are rapidly changed, either by design or as a result of some unforeseen event. The rate of change of the system boundary conditions determine the severity of the flow parameter changes that result. In many cases the resulting pressures generated in the flow system determine the design conditions.

The most well documented area of transient analysis refers to the propagation of pressure surges through full bore flow carrying pipe systems. For example the pressure variations following inadvertent power failure to a pipeline pumping station often provides the system design condition and leads to the need to introduce safety systems such as air chambers, increased pump inertia or by pass systems. Similarly the surges generated by load rejection by hydroelectric power station turbines lead to surge propagation in the water supply tunnels, normally dealt with by the inclusion of surge shafts cut into the rock surrounding the hydroplant supply tunnels.

The vast majority of research into unsteady flow has therefore been directed towards the large civil engineering applications such as power stations, cross country oil, water or sewage pipelines. Currently these areas are well documented and several texts are available describing appropriate analysis and design techniques [1,2].

The prediction techniques currently employed may be traced back to the late 19th century and for many years were based on the Bergeron graphical methods, that incidentally also find application in electrical surge problems.

The widespread availability of digital computers from the 1960's introduced or made practical, more versatile methods. Although translations of the graphical techniques were used initially, the numerical method of characteristics employed to solve the differential equations defining the unsteady flow has now come to be widely accepted as the more versatile technique [3].

As mentioned the applications to be found in the literature refer almost exclusively to large scale civil engineering applications. However, the problem of unsteady flow translates across the boundaries of scale, being dependent on the relationship between the rate of change of boundary conditions and the propagation rate of transient pressures within the system. Such applications may be found in a wide range of small scale systems, including, for example, diesel fuel injection systems. The analysis technique based on the method of characteristics solution was successfully applied to a transient analysis of the Concorde fuel system, both in the ground refuelling mode and the in flight fuel transfer and emergency ejection modes [4]. Similarly the methodology has been applied to airport hydrant refuelling networks.

Although this vast bank of experience and literature exists it is, as mentioned, primarily directed to full bore flow large civil engineering applications.

Unsteady flow may, by definition, also occur in partially filled pipe or open channel flow. Again the available literature applies to the large excavated straight sided channels to be found in such applications as power station turbine discharge channels. In the open channel flow situation the full bore flow transient pressure changes are replaced by channel depth variations, pressure waves moving at the appropriate sonic velocity being replaced by surface waves. The simplification possible in air or vapor free fluid full bore flow that the wave propagation speed is constant, depending only on the liquid and pipe materials and dimensions, no longer holds as the wave propagation speed depends on flow depth and channel shape. Thus an extra variable is effectively added to the local flow velocity and pressure, or depth, namely the local wave speed.

As will be seen in the analysis presented this also introduces other difficulties in the numerical solution of the governing equations since the uniform x-t grid system generally employed has to be modified to allow for variable characteristic slope as the local wave speed varies.

As with the scale translation from hydroelectric plant to aircraft fuel system, so too does the basic flow analysis translate from large excavated open channels to partially filled drain piping systems. The boundary conditions change to represent, for example, varying inflow dependent on the appliances connected to the system. The channel cross sectional shape does not constrain the application of the fundamental analytical approach.

At present no numerical analysis technique has been documented to predict the depth and flow rate along gravity driven unsteady flow in partially filled drainage sized systems; however the analysis techniques incorporating the method of characteristics together with boundary conditions expressed in terms of time dependent depth or flow rates may be employed.

The current report is designed to act as an introduction to the use of these methods to predict depth-time profiles along a simulated drain. In particular the analysis will attempt to predict the depth history upstream of a deposited solid in the drain pipe as a prelude to a consideration of the forces acting on the solid and its subsequent motion along the drain pipe.

In addition the analysis techniques are shown to be applicable to determine flow attenuation in long drain pipes, this topic being considered increasingly important due to the probable reduction in overall system flow rates due to water conservation proposals.

Similarly the techniques can be applied to pipe sizing calculations in order to avoid the occurrence of local full bore flow and associated venting problems.

The differential equations defining unsteady flow are presented in this report, together with the application of the method of characteristics to yield a total differential equation which may be solved by numerical methods. The method of specific time intervals is presented together with the necessary formulation of boundary equations to represent variable inflow, pipe discharge and leakage flow past the solid. Finally a computer program designed to analyze the transient depth response to a stationary solid in a pipe supplied with a variable inflow is presented together with typical output for flow and system variables representative of drainage design values.

Extension of the methods described to flow attenuation and solid motion prediction are also proposed.

2. UNSTEADY FLOW IN OPEN CHANNELS

2.1 DEFINING EQUATIONS

The equations defining one dimensional unsteady flow are presented for open channels. Frictional losses are assumed to be adequately represented by the Manning equation utilizing the local velocity and flow depth. Channel slopes are assumed small enough for cosine slope ($\cos\alpha$) to be approximately 1.

Figure 1 illustrates an elemental fluid strip. Application of the unsteady momentum equation yields:

- | | | |
|-------|---------------------------------------|---|
| (i) | Net hydrostatic force: | $\rho g \frac{\partial h}{\partial x} \Delta x A$ |
| (ii) | Shear force on wetted:
perimeter P | $-\tau_o P \Delta x$ |
| (iii) | Gravity force in flow:
direction | $\rho g A \Delta x \sin\alpha$ |
| (iv) | Net efflux momentum: | $\frac{\partial}{\partial x} (\rho V^2 A) \Delta x$ |
| (v) | Increase of momentum within: | $\frac{\partial}{\partial t} (\rho AV) \Delta x$ |

Thus the momentum equation becomes

$$\begin{aligned}
 -\rho g \Delta x A \frac{\partial h}{\partial x} - \tau_o P \Delta x + \rho g A \Delta x \sin\alpha \\
 = \frac{\partial}{\partial x} (\rho V^2 A) \Delta x + \frac{\partial}{\partial t} (\rho AV) \Delta x
 \end{aligned}
 \tag{1}$$

Expanding and dividing by $\rho A \Delta x$

$$g \frac{\partial h}{\partial x} + \frac{\tau_o}{\rho m} - g \sin \alpha + 2V \frac{\partial V}{\partial x} + \frac{V^2}{A} \frac{\partial A}{\partial x} + \frac{V}{A} \frac{\partial A}{\partial t} + \frac{\partial V}{\partial t} = 0 \quad (2)$$

where $m = A/P$

The continuity equation applied to the control volume of Figure 1 yields

$$\frac{-\partial (\rho AV) \Delta x}{\partial x} = \frac{\partial (\rho A \Delta x)}{\partial t} \quad (3)$$

Expanding and dividing by $\rho \Delta x$

$$V \frac{\partial A}{\partial x} + \frac{\partial A}{\partial t} + A \frac{\partial V}{\partial x} = 0 \quad (4)$$

Multiplying (4) by V/A and subtracting from equation (2) yields

$$g \frac{\partial h}{\partial x} + \frac{\tau_o}{\rho m} - g \sin \alpha + \frac{V \partial V}{\partial x} + \frac{\partial V}{\partial t} = 0 \quad (5)$$

Let $S_o = \sin \alpha$ and $gS = \tau_o / \rho m$

where S is the slope of the energy grade line as defined by the Manning equation

$$S = \frac{n^2 V^2}{m^{4/3}} \quad (6)$$

Note that $S = S_o$ only under steady uniform flow conditions.

Equation 5 becomes

$$g \frac{\partial h}{\partial x} + g(S - S_o) + \frac{V \partial V}{\partial x} + \frac{\partial V}{\partial t} = 0 \quad (7)$$

From Figure 1 $\partial A = \partial h \cdot T$.

where T is surface width.

$$\text{Hence } \frac{\partial A}{\partial x} = T \frac{\partial h}{\partial x},$$

$$\text{and } \frac{\partial A}{\partial t} = T \frac{\partial h}{\partial t}$$

so that equation 4 becomes

$$VT \frac{\partial h}{\partial x} + T \frac{\partial h}{\partial t} + A \frac{\partial V}{\partial x} = 0 \quad (8)$$

Thus equations 7 and 8 represent the unsteady flow conditions in terms of local depth and average velocity. These equations may be solved by means of the method of characteristics.

2.2 SOLUTION BY METHOD OF CHARACTERISTICS

Equation 7 and 8 may be combined as

$$L_1 + \lambda L_2 = 0$$

where L_1 is equation 7 and L_2 equation 8

$$\left[\frac{\partial V}{\partial x} (V + \lambda A) + \frac{\partial V}{\partial t} \right]_1 + \lambda T \left[\frac{\partial h}{\partial x} (V + \frac{g}{\lambda T}) + \frac{\partial h}{\partial t} \right]_2 + g(S - S_0) = 0 \quad (9)$$

In order to solve equations 7 and 8 it is necessary to transform the equations into a total derivative expression.

For the terms in bracket [1] to become the total derivative dV/dt , it is necessary for

$$V + \lambda A = \frac{dx}{dt}$$

and for the terms in bracket [2] to become the total derivative dh/dt then

$$\frac{dx}{dt} = V + g/\lambda T$$

$$\text{hence } V + \lambda A_- = V + g/\lambda T$$

$$\text{therefore } \lambda = \pm \sqrt{\frac{g}{AT}}$$

$$\text{and } \frac{dx}{dt} = V \pm \sqrt{\frac{gA}{T}}$$

The term $\sqrt{\frac{gA}{T}}$ has dimensions of velocity and is identified as the local wave speed

$$c = \sqrt{\frac{gA}{T}} \quad (10)$$

Equation 9 may now be expressed as:

$$\frac{dV}{dt} \pm \frac{g}{c} \frac{dh}{dt} + g(S - S_o) = 0 \quad (11)$$

$$\text{subject to } \frac{dx}{dt} = V \pm c \quad (12)$$

Referring to Figure 2, if the variables V and h are known at R and S then four equations may be written in terms of the unknowns at point P, namely

$$C^+ \left[V_P - V_R + g \int_{h_R}^{h_P} \frac{1}{c} dh + \int_{t_R}^{t_P} g(S - S_o) dt = 0 \right] \quad (13)$$

$$\left[x_P - x_R = \int_{t_R}^{t_P} (V+c) dt \right] \quad (14)$$

$$C^- \left[V_P - V_S + g \int_{h_S}^{h_P} \frac{1}{c} dh + \int_{t_S}^{t_P} g(S - S_o) dt = 0 \right] \quad (15)$$

$$\left[x_P - x_S = \int_{t_S}^{t_P} (V-c) dt \right] \quad (16)$$

It is stressed that these equations are paired, i.e. equation 13 only holds if equation 14 is satisfied and 15 only applied if 16 is satisfied. Generally these are referred to as C^+ and C^- characteristics as shown above.

In most cases a first order integration is satisfactory, however attention must be paid to the choice of time step to ensure a stable solution, as mentioned by Fox [1].

In terms of Figure 2 the time step must be sufficiently small to ensure that points R and S fall within $\pm \Delta x$ on either side of point P.

Generally a suitable rule is to set

$$\Delta t = \Delta x / 2c \quad (17)$$

where c is the wave speed appropriate to the initial flow in the channel prior to the initiation of unsteady flow. As will be seen the solution requires an initial steady flow in the channel, although this may be infinitesimal.

Applying a first order approximation to equations 13 to 16, in terms of Figure 2 yields

$$V_P - V_R + \frac{g}{c_R} (h_P - h_R) + g(S_R - S_0)\Delta t = 0 \quad (18)$$

$$x_P - x_R = (V_R + c_R) \Delta t \quad (19)$$

$$V_P - V_S - \frac{g}{c_S} (h_P - h_S) + g(S_R - S_0)\Delta t = 0 \quad (20)$$

$$x_P - x_S = (V_S - c_S) \Delta t \quad (21)$$

It will be noted that conditions at R and S at time $(t-\Delta t)$ are determined by interpolation between points A and C and C and B respectively. This interpolation introduces a dispersive element to the calculation as effects arriving at A, C and B at time $t - \Delta t$ are assumed, via the interpolation, to effect conditions at R and S at time $t - \Delta t$. This is comparable to increasing the local wave speed.

Interpolation effects should be minimized by arranging for R and S to fall as close as possible to A and B by adjusting the time step Δt .

Two flow regimes may be identified for open channels defined in terms of the Froude Number $Fr = V/\sqrt{gh}$

1) Subcritical flow, Froude $N^0 < 1$

Here the local wave speed is greater than the flow average velocity. Thus waves may be propagated upstream.

2) Supercritical flow, Froude $N^0 > 1$

Here the local wave speed is less than the average flow velocity at that section, hence waves may not be propagated upstream. Flow may be transformed from supercritical to subcritical via a hydraulic jump.

Figure 3 illustrates the importance of these two flow regimes on the solution of equations 13 to 16.

If $c > V$ then the conditions at P are determined by the intersection of the C^+ and C^- drawn from P into the AC and BC sections.

If $c < V$ then conditions in the downstream section BC cannot effect point P. The slope of the C^- characteristic, PS, becomes positive and both R and S lie in the AC section as shown.

For the subcritical flow encountered in the drainage applications considered and the equations derived below refer to this flow regime. Similar derivations may be undertaken for supercritical flow.

Referring to Figure 2 for subcritical flow:

$$\frac{V_C - V_R}{V_C - V_A} = \frac{x_C - x_R}{x_C - x_A} = (V_R + c_R) \frac{\Delta t}{\Delta x}$$

$$\frac{c_C - c_R}{c_C - c_A} = \frac{x_C - x_R}{x_C - x_A} = (V_R + c_R) \frac{\Delta t}{\Delta x}$$

and
$$\frac{h_C - h_R}{h_C - h_A} = (V_R + c_R) \frac{\Delta t}{\Delta x}$$

as $x_P = x_C$ and $x_P - x_R = (V_R + c_R)\Delta t$

Solution yields

$$V_R = \frac{V_C + \theta (-V_C c_A + c_C V_A)}{1 + \theta (V_C - V_A + c_C - c_A)} \quad (22)$$

$$c_R = \frac{c_C(1 - V_R \theta) + c_A V_R \theta}{1 + c_C \theta - c_A \theta} \quad (23)$$

$$h_R = h_C - (h_C - h_A)(\theta(V_R + c_R)) \quad (24)$$

Similarly

$$V_S = \frac{V_C - \theta(V_C c_B - c_C V_B)}{1 - \theta(V_C - V_B - c_C + c_B)} \quad (25)$$

$$c_S = \frac{c_C + V_S \theta(c_C - c_B)}{1 + \theta(c_C - c_B)} \quad (26)$$

$$h_S = h_C + \theta(V_S - c_S)(h_C - h_B) \quad (27)$$

The determination of conditions at P at time $t + \Delta t$ requires the following steps:

- (i) All conditions known at time t for nodal points A B C etc.
- (ii) Values of V , h and c at interpolation points R and S calculated from equations 21 - 27.
- (iii) Using these values of V , h , and c the conditions at P, i.e. velocity V and depth h , at time $t + \Delta t$ are calculated by means of equations 18 and 20.
- (iv) The value of wave speed c at P at time $t + \Delta t$ is calculated from equation 10. The value of flow surface width and cross sectional area are calculated from flow depth, h , and the channel shape relationships.
- (v) The sequence is repeated at each time step.

2.3 APPLICATION OF SOLUTION TO DRAINAGE FLOW

Equations 18 to 21 may be expressed as:

$$\begin{aligned} V_P &= X_2 - X_1 h_P \\ x_P - x_R &= (V_R + c_R)\Delta t \end{aligned} \quad C^+ \quad (28)$$

$$\begin{aligned} V_P &= X_4 + X_3 h_P \\ x_P - x_S &= (V_S - c_S)\Delta t \end{aligned} \quad C^- \quad (29)$$

where $X_1 = g/c_R$

$$X_3 = g/c_S$$

$$X_2 = V_R + g h_R - g(S_R - S_o)\Delta t$$

$$X_4 = V_S - g h_S - g(S_S - S_o)\Delta t$$

Figure 4 illustrates a typical drainage pipe length to be analyzed in terms of flow depth and velocity at each section.

The application may be dealt with in two distinct sections:

(i) Internal or nodal points.

The values of h , V and c at all points Δx apart between $x = \Delta x$ and $x = (L - \Delta x)$ may be calculated by the sequence set out above, namely by simultaneous solution of equations 28, 29 and use of equation 10 to yield wave speed.

(ii) Boundary conditions.

In order to predict h , V and c at the system boundary it is necessary to solve either 28 or 29 with an appropriate boundary condition.

At pipe entry a suitable boundary condition would be the inflow profile

$$Q = f(t)$$

to be solved with the C^- characteristic:

$$Q(t) = V_1 A_1 = f(t)$$

$$V_1 = X_4 + X_3 h_1$$

$$Q(t) = A_1 (X_4 + X_3 h_1)$$

where $A_1 = f(h_1)$

$$Q(t) = f(h_1)(X_4 + X_3 h_1)$$

In the form -

$$0 = Q(t) - f(h_1) (X_4 - X_3 h_1) \quad (30)$$

this relationship may be solved by bisection method as values of Q are known at each time step. The channel cross section shape relationship is also required so that no direct solution of equation 30 is available for circular section channels.

At pipe exit in the subcritical flow regime the flow depth approaches the critical depth value, given by zero value of the expression:

$$\frac{Q^2}{g A_{crit}^3} T_{crit} = 1,$$

where A and T are functions of depth, h .

This condition may be solved with the C^+ characteristic

$$V_{N+1} = X_2 - X_1 h_{N+1}$$

where $N = N^\circ$ of pipe length sections, each of length Δx .

The boundary condition becomes

$$[(X_2 - X_1 h_{N+1}) A_{N+1}]^2 T_{N+1} / g A_{N+1}^3 - 1 = 0 \quad (31)$$

Solution may again be achieved by use of the bisection method together with the use of the area to depth relationship for the channel.

2.4 APPLICATION OF WASTE SOLID BOUNDARY CONDITION

Considering a stationary solid deposited at some point along the waste pipe, the water depth and velocity upstream of the solid may be predicted if a suitable boundary equation may be written linking flow past the solid to upstream conditions.

Figure 5 illustrates the relationship between flow past a stationary solid and the specific energy upstream. These results were compiled during a Brunel University Drainage Research Group study of solid transport in drainage systems.

From the experimental work summarized by Figure 5, the flow past the solid may be expressed by the following relationship:

$$Q = K(h + \frac{V^2}{2g} - SE_0)^2 \quad (32)$$

where $SE = h + V^2/2g$, flow specific energy and SE_0 is the flow specific energy required for flow initiation past the solid (when $SE_0 \neq 0$)

Equation 32 may then be solved with the C^+ characteristic

$$V_{N+1} = X2 - X1 h_{N+1}$$

where $Q = V_{N+1} A_{N+1}$

so that

$$A_{N+1} (X2 - X1 h_{N+1}) = K [h_{N+1} + \frac{1}{2g} (X2 - X1 h_{N+1})^2 - SE_0]^2$$

This expression results in a quartic in terms of water depth upstream of the solid, h_{N+1} , see Appendix 2.

This quartic must be solved by an iterative technique as the flow area, A_{N+1} , is a function of h_{N+1} .

The Newton-Raphson method may be employed to carry out the necessary iteration solution.

Once the value of h_{N+1} has been determined the value of V_{N+1} and C_{N+1} may be determined from equation 28 and 10.

As mentioned the SE_0 term is the flow specific energy required to initiate flow past the stationary solid. If the value of flow specific energy at time t is less than SE_0 then the value of flow velocity at the solid at time $t + \Delta t$ is set equal to zero. The flow depth then comes directly from equation 28 as:

$$h_{N+1} = X2/X1 \quad (33)$$

This implies that the flow depth upstream of the solid must rise to SE_0 prior to the initiation of flow past the solid.

This solution is set out in detail in Appendix 2.

In this model no account is taken of flow downstream of the solid.

From Figure 5 the flow past the solid may alternatively be expressed as

$$Q = K_1 (SE_U - SE_D)^2$$

where SE_U , SE_D are the specific energy values immediately upstream and downstream of the solid.

This boundary equation may be used to link the flow conditions upstream and downstream of the solid, for example the upstream flow is governed by the C^+ equation 28 and that downstream is governed by the C^- equation 29.

This formulation of the boundary condition would allow a series of nodal points downstream of the solid to be dealt with, the pipe being then terminated by the boundary condition already described, i.e. critical depth at discharge.

The techniques described above have been included in a Fortran program TRANSCA run on the NBS Center for Building Technology Perkin Elmer 732 computer.

This program is included in Appendix 1 together with a flow chart and description and sample input data while representative program output are included in Appendix 3.

3. PROGRAM OUTPUT AND DEVELOPMENT

Figures 6, 7 and 8 illustrate the predicted depth and flowrate at sections along the simulated drain pipe considered, while Appendix 3 presents typical program output.

The program calculation technique requires a known steady uniform flow to be set up along the channel prior to the calculation of unsteady conditions. In the model presented this is assumed to be established with no solid in the pipe. The calculation then assumes the presence of the solid from the first time step onwards. This explains the depth increasing wave that is shown in Figure 8 moving upstream from the solid boundary. This is merely a function of the initial conditions chosen.

An alternative base condition would be either steady flow with the solid in place, with the associated back water profile upstream of the stationary solid, or stationary fluid trapped behind the solid. In both cases the fluid depth would increase at the solid location, forming the expected backwater curve profile.

The former set of flow conditions could have been achieved in the current simulation by holding the inflow rate constant, however this would have required a longer computer run. The establishment of steady conditions is demonstrated in Figure 6 as the inflow profile is assumed to become constant.

The results demonstrate that the techniques described may be employed to yield values of depth and flowrate under unsteady flow conditions in open channel flow. In particular the results indicate that the concept and use of specific energy to leakage flow characteristic, rather akin to a valve pressure flow relationship, is capable of accurately describing the boundary conditions at a stationary solid. The values of depth at the solid with time obviously require experimental verification, however the depths predicted are generally in line with observations of water build up behind stationary solids made during the Brunel University drainage investigations.

Two main development paths may be proposed based on the analysis techniques described:

(i) Application to the moving solid case.

This case would require the introduction of flow relative velocity into the boundary conditions describing the solid described above. It is proposed that, once this has been done, the stationary solid specific energy differential across the blockage to flow past relationship is assumed to apply to the moving case.

A number of unknowns are required to obtain a solution, namely the force balance necessary to be overcome to initiate motion and the solid-flow momentum equation that would determine solid velocity and distance traversed in each time step.

One simplifying assumption that could be made would be to assume that the flow ahead of the solid, i.e. downstream, assumes the normal depth and velocity characteristic to the channel dimensions and the flow past the solid.

This case is currently being considered.

(ii) Application of the techniques described to the prediction of flow attenuation in long channels.

At first sight it would appear that the existing program would be capable of dealing with this case, however the errors due to interpolation made necessary by the adoption of a fixed time-distance grid, Figure 2, could introduce errors. The interpolation techniques effectively disperse the moving wave fronts and would tend to over-estimate the attenuation produced.

A free grid in which both time and distance become calculated variables is a solution to this problem. This is not normally done as it results in a rather "untidy" output, however this method will be investigated as part of the current investigation.

As mentioned the method of characteristics requires that there be an initial steady flow condition along the channel, or that the flow is at rest at known depth. This limiting condition is necessary to provide the base values of depth, velocity and wave speed at each calculating position. The influence of magnitude of this initial assumed flow on the attenuation of a constant superimposed inflow will also be considered as part of (ii) above, together with an assessment of the effect of the interpolation ratio employed in choosing the appropriate calculation time step.

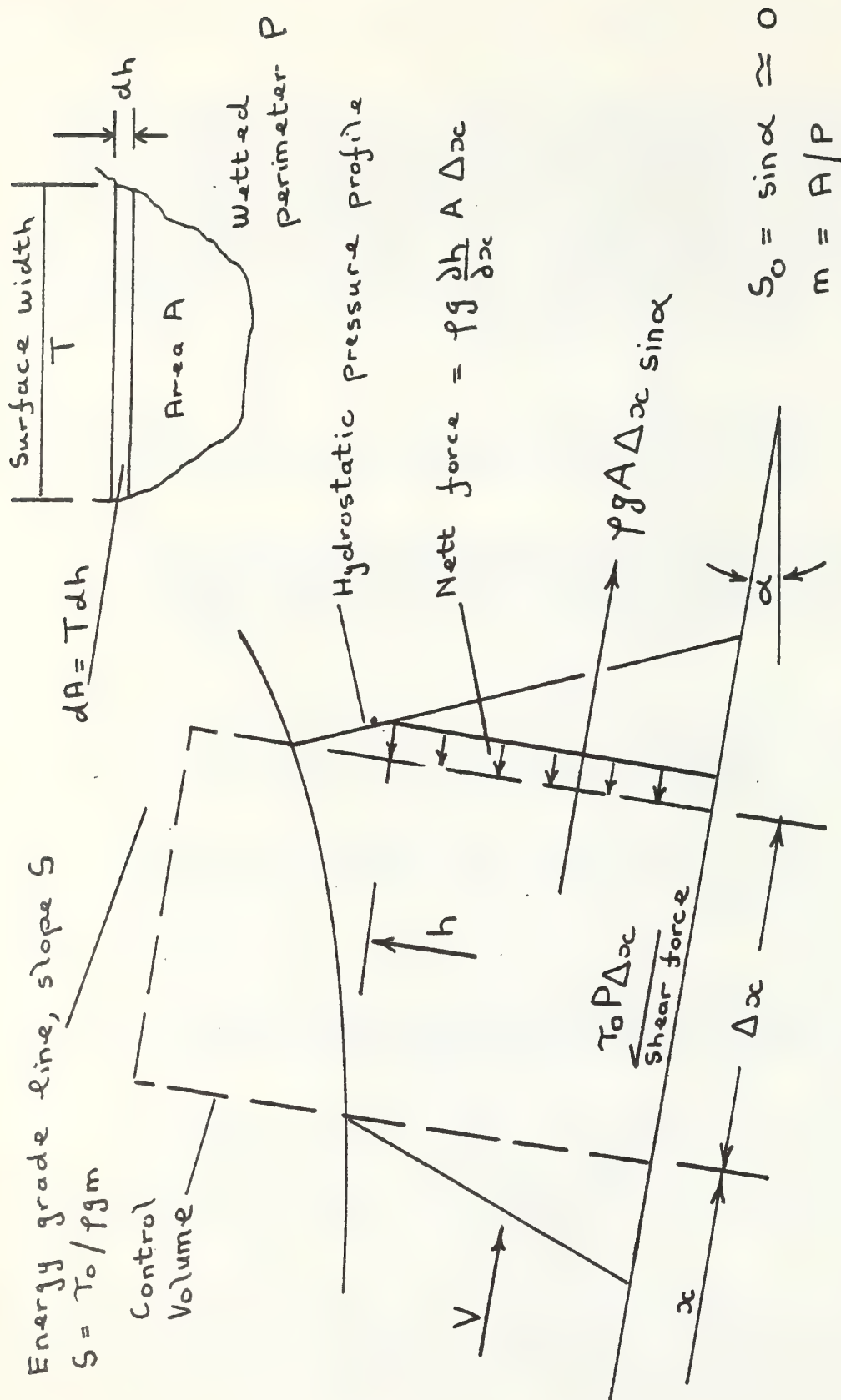
4. CONCLUSIONS

The program presented indicates that the techniques involved in the solution of unsteady flow problems via the method of characteristics may be applied to partially filled drainage pipeflow.

The methods presented will be employed to investigate further their application to solid waste transport analysis and to the study of flow attenuation in long channels, typically representative of underground building drainage systems.

5. REFERENCES

1. J. A. Fox, Hydraulic analysis of unsteady flow in pipe networks. Macmillan Press London, 1977.
2. Wylie, E. B. and Streeter V. L, Fluid Transients, McGraw Hill, New York 1978.
3. Douglas, J. F., Gasiorek, J. M., and Swaffield, J. A., Fluid Mechanics, Pittman, London, 1979.
4. Swaffield, J. A., Application of the method of characteristics to a pressure transient analysis of the BAC/SNIAS Concorde fuel system. Proceedings Institute of Mechanical Engineer London, 1972.



Note 2nd order terms neglected

Figure 1. Application of momentum equation to unsteady flow in a general open channel.

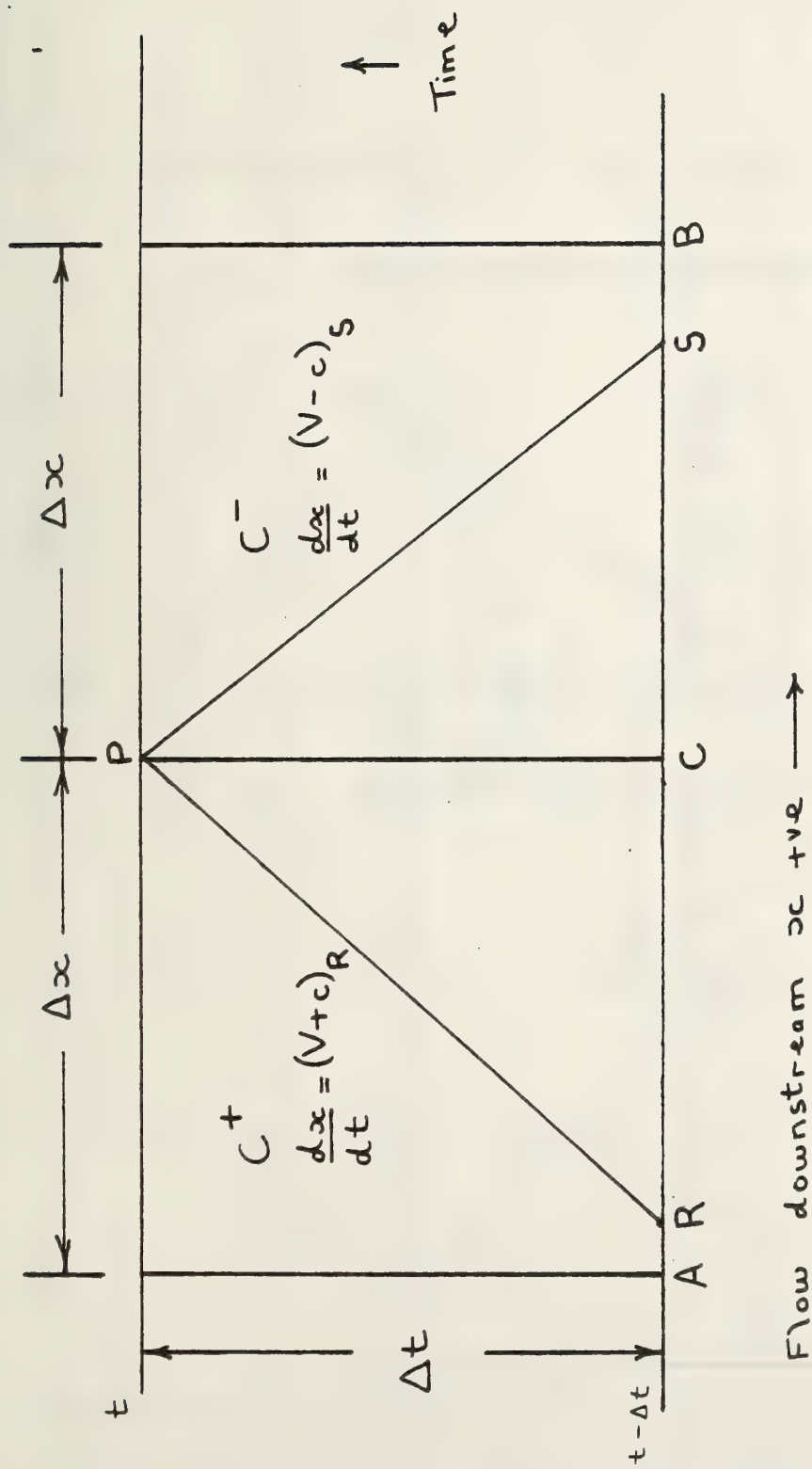


Figure 2. Specified time interval technique, illustrating necessity for base condition interpolation.

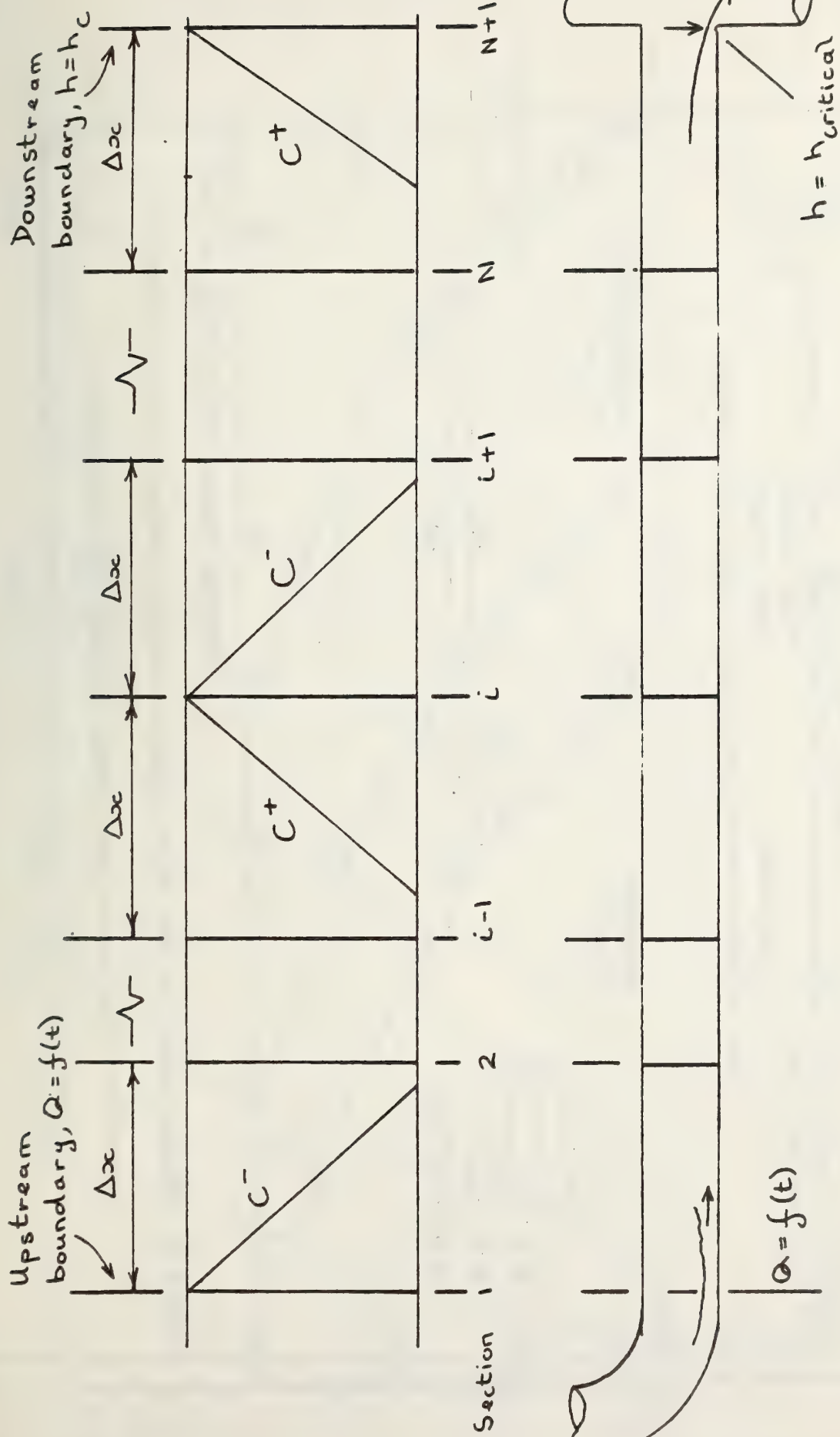


Figure 4. Application of specified time interval grid to simple channel with known entry and exit boundary equations.

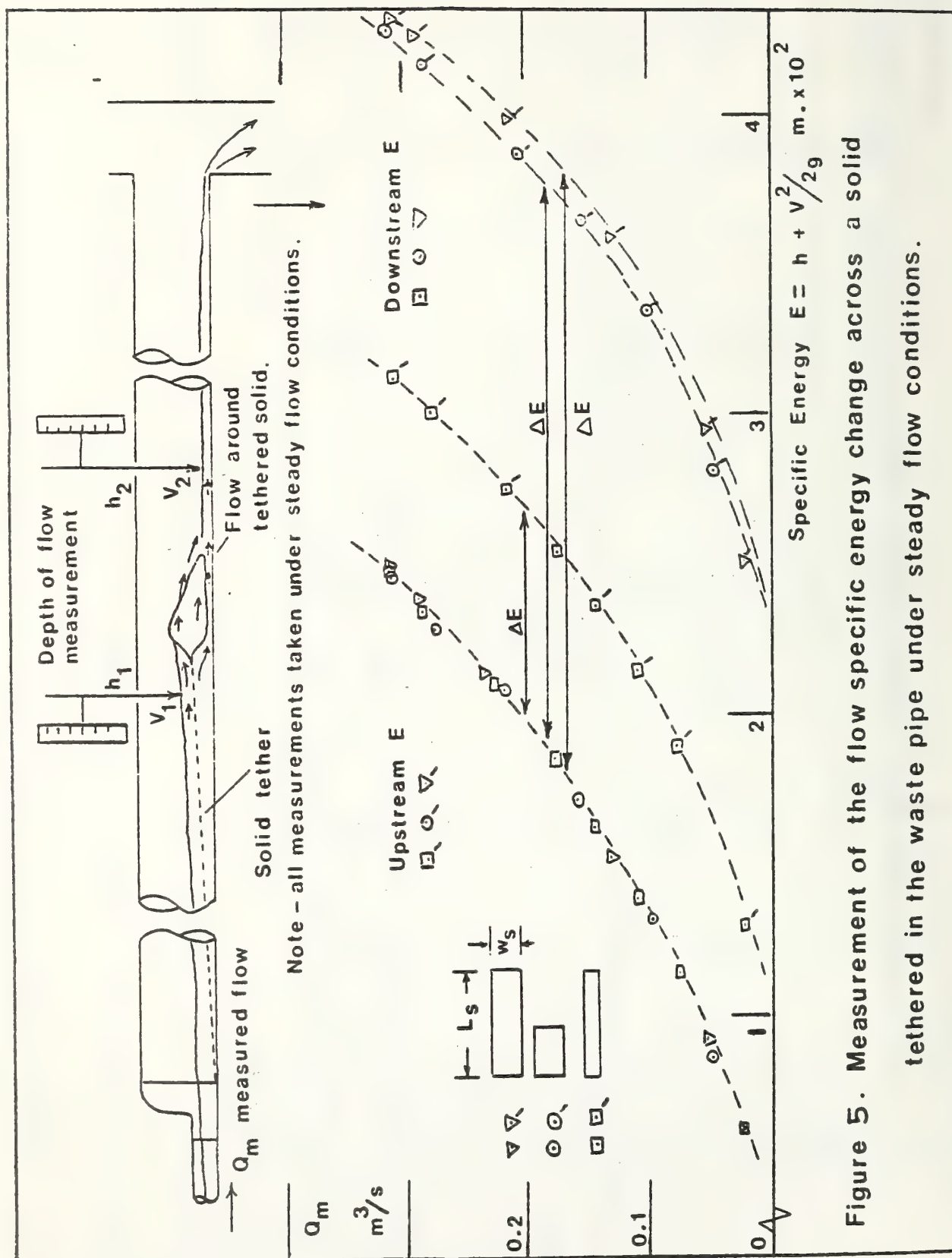


Figure 5. Measurement of the flow specific energy change across a solid tethered in the waste pipe under steady flow conditions.

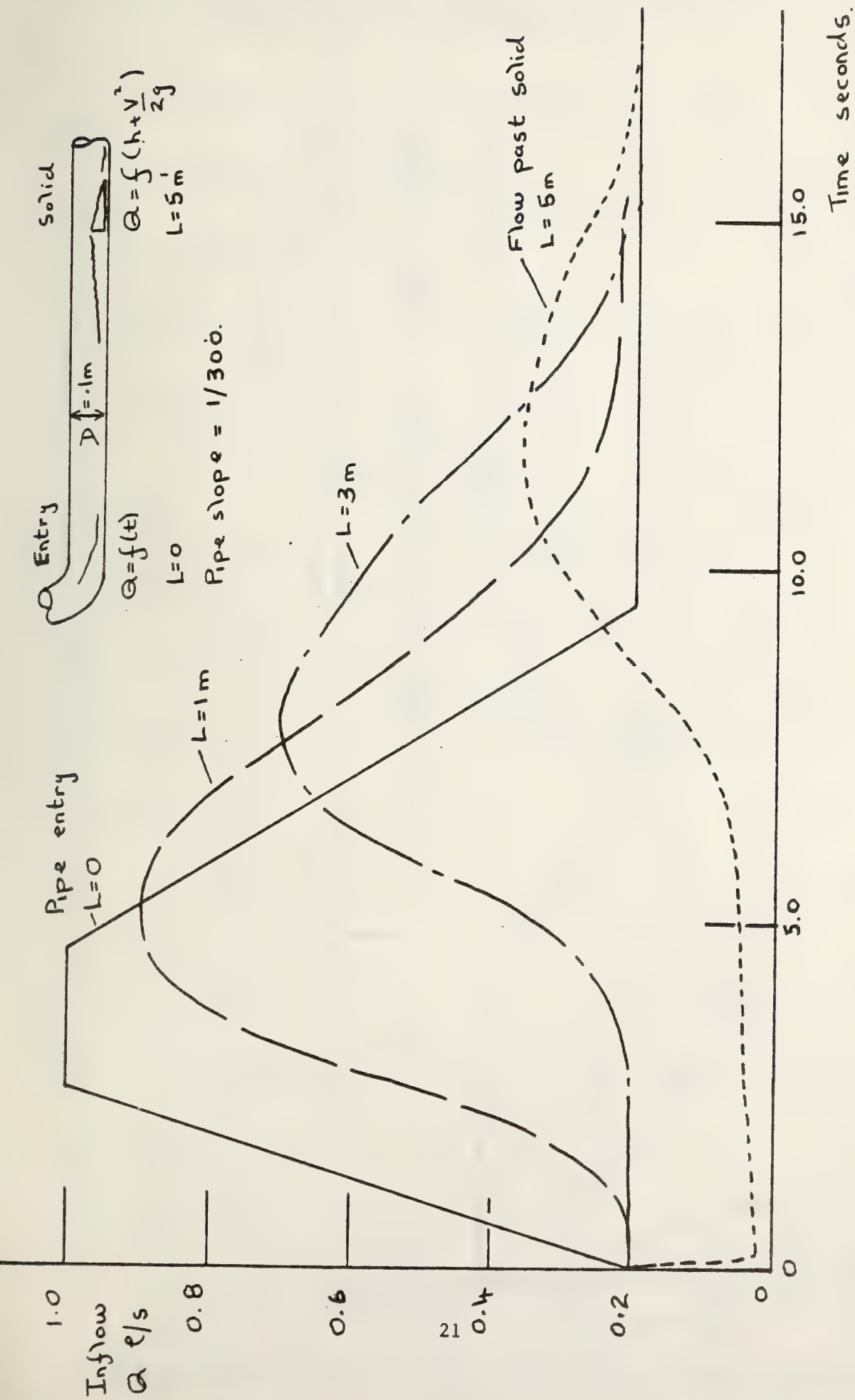


Figure 6. Flowrate vs time profiles at pipe entry, solid boundary and intermediate points along the simulated drain pipe.

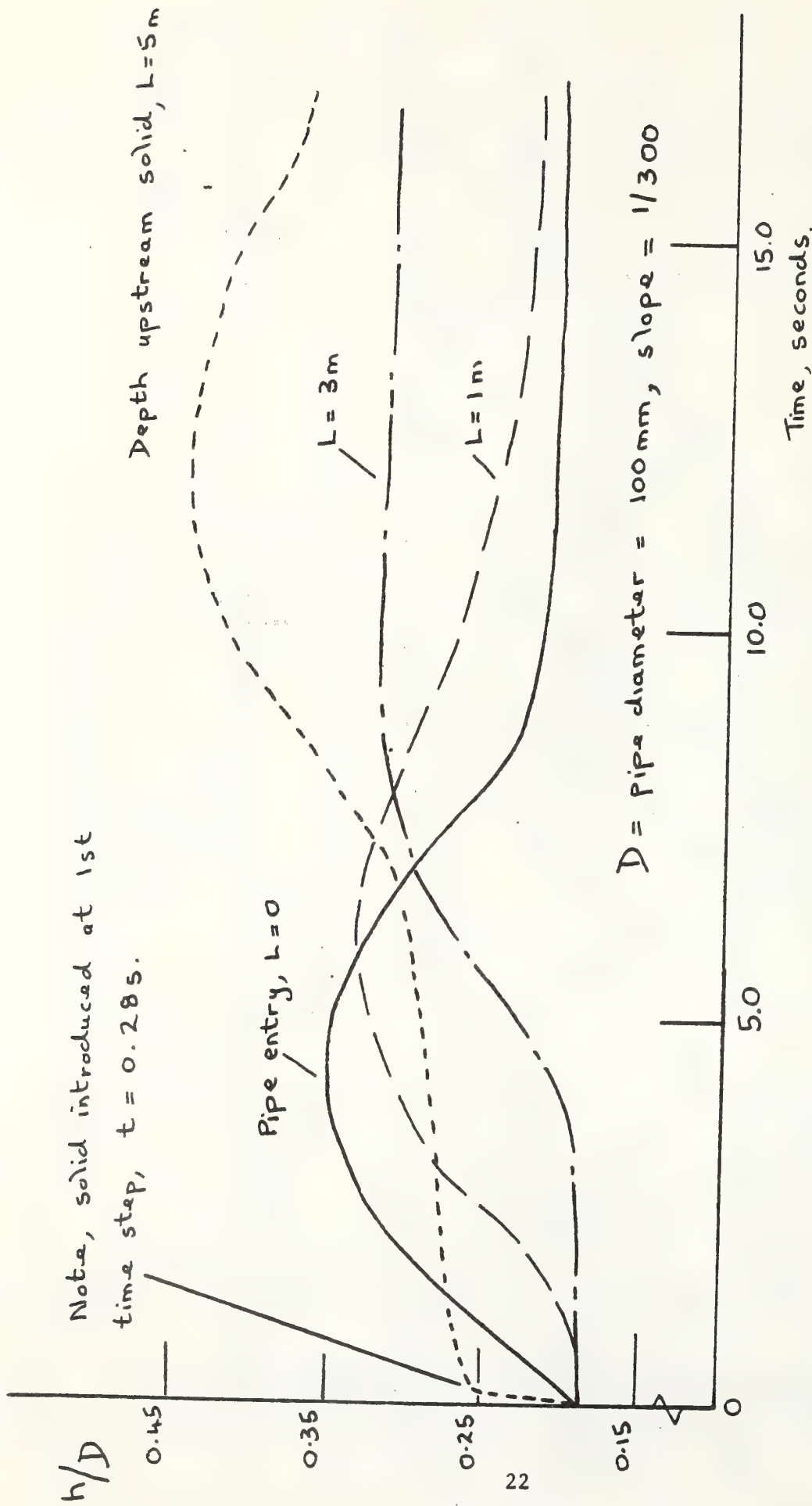


Figure 7. Depth vs time profiles along the simulated drainage pipe following an inflow variation.

Figure 8. Depth profiles along the 5m simulated drain at 14 time intervals illustrating surface wave motion.

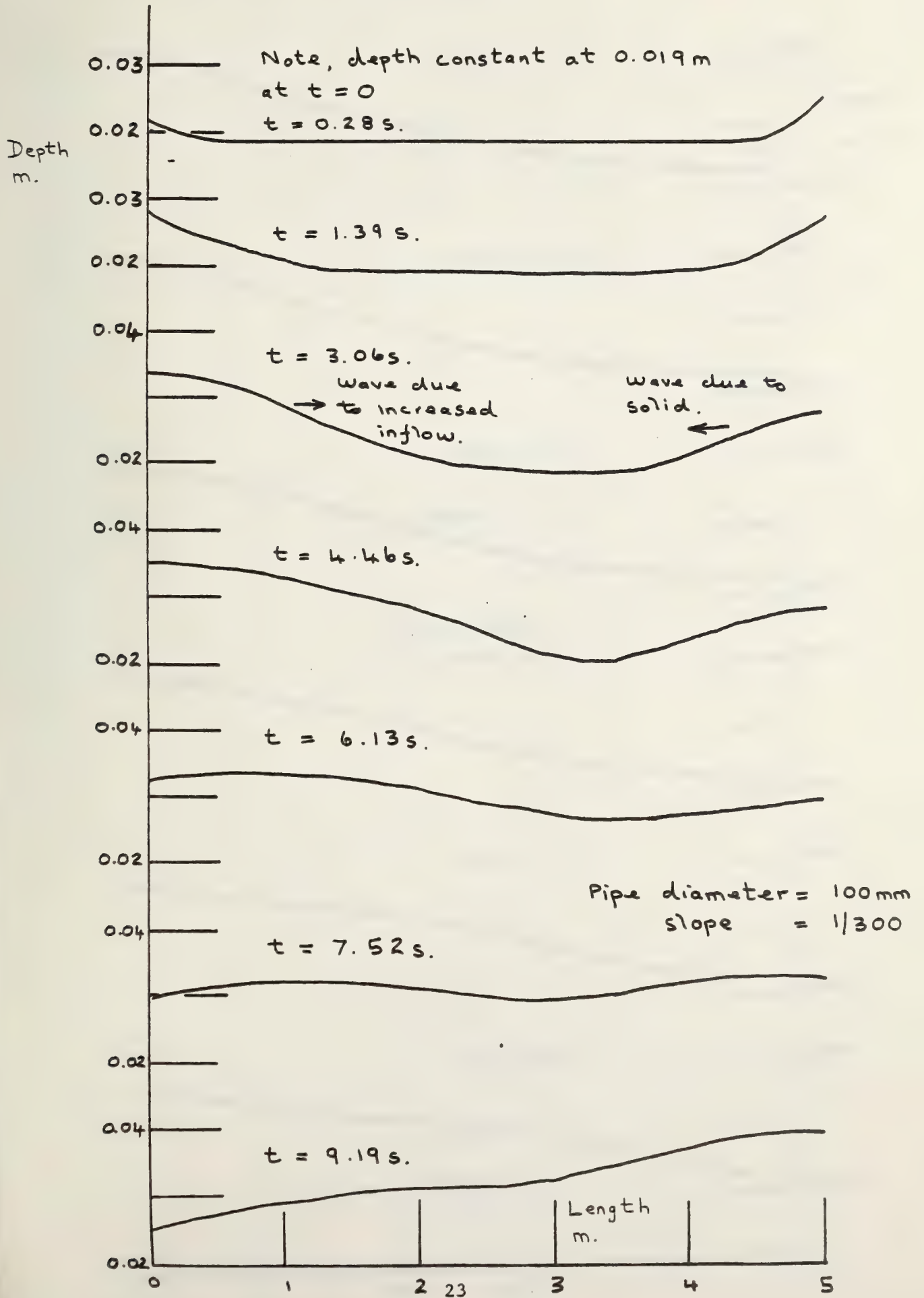
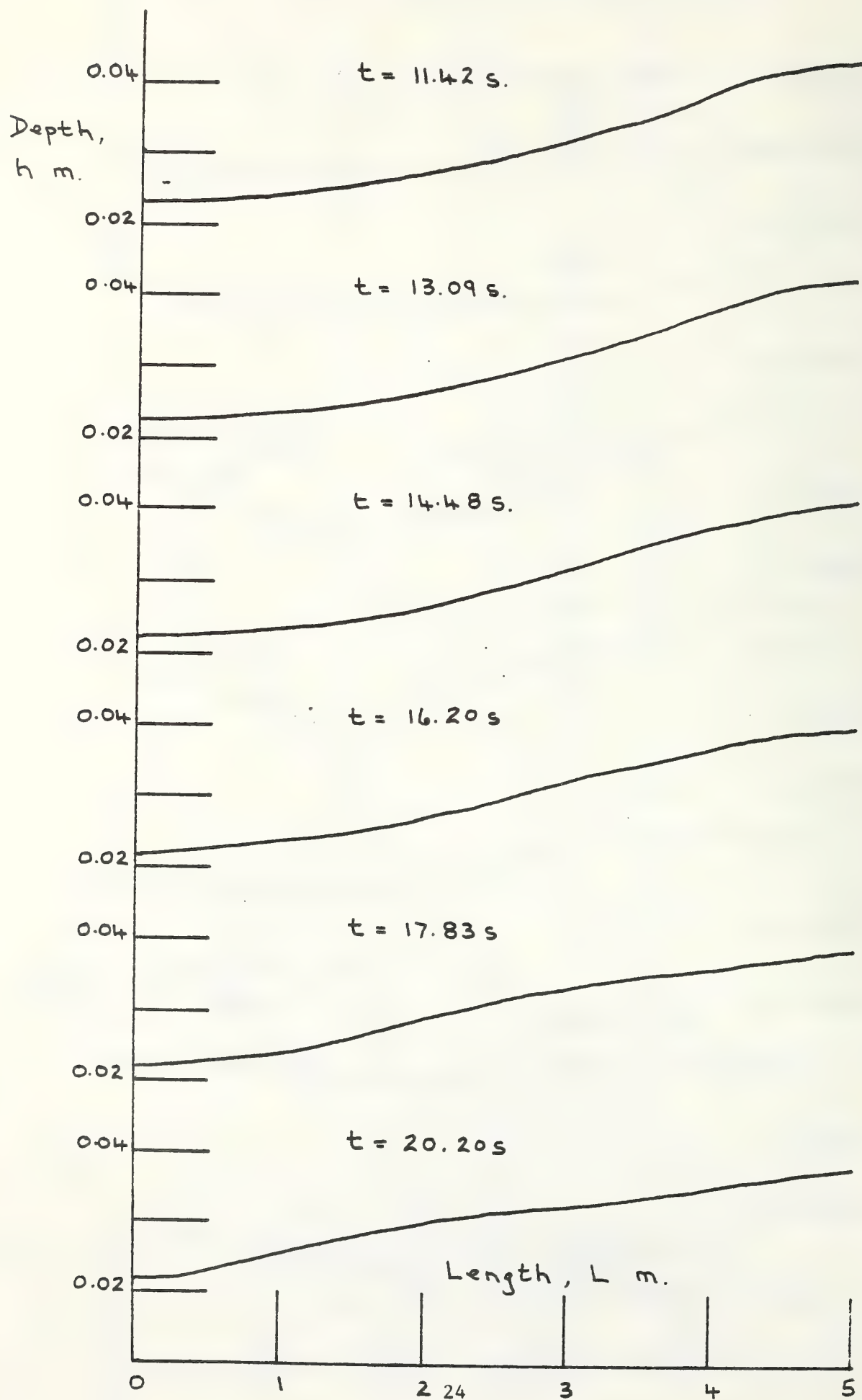


Figure 8 cont. Depth profiles along the 3m simulated drain at 14 time intervals illustrating surface wave motion.



Appendix 1

Program TRANSCA

Program TRANSCA

This appendix presents a complete print out of this program, written in Fortran, together with a flow chart and sample input data. The program was run on the NBS Center for Building Technology Perkin Elmer 732 computer.

It was found necessary to employ double precision calculating techniques in the program due to the inclusion of calculations involving the root of small differences between large numbers. Otherwise no special facilities are required.

The program accepts data in SI units with the exception of the inflow profile, this is read in as litres/second and corrected to m^3/s within the program.

FLOW CHART PROGRAM TRANSCA.

Set up initial conditions:

Time = 0.0

Read pipe data, diameter D , Manning coeff. RM ,
slope $S\phi$, length PL .

Read solid data, $SE\phi$ - min. specific energy to
establish flow past solid.

XK - coeff. in $Q = XK(SE - SE\phi)^2$
solid characteristic.

Read calculation data, N - N^o pipe sections,
 $TMAX$ - duration calculation
 $TFAC$ - time step factor

Read inflow profile, $NPTS$ - N^o of coordinate pairs
assumed linked by straight line
Flow - Q_{IN} , time TIN .

Adjust flow from l/s data to m^3/s for calc.



Calculation of initial steady uniform
conditions at time zero

CALL DEPTH - calc. flow normal depth HN .

CALL SHAPE - calc. flow area, A , width, T .

CALL WAVESPD - calc. initial wave speed CN

Calc. steady state loss from Manning eqn.



Assign time zero values along pipe.

$$VP = QIN/A, \quad QP = QIN,$$

$$HP = HN, \quad CP = CN$$

↓
Set up interpolated points R, S to serve as base conditions for next time step

$$HR = HS = HP, \quad VR = VS = VP,$$

$$CR = CS = CP, \quad SR = SS = SL \emptyset.$$

↓
CALL ASSIGN - equate HP-H, VP-V, CP-C

↓
Calculation of time step DT

$$DX = PL/N, \quad DT = DX / (TFAC * CN)$$

↓
WRITE Initial conditions and descriptions
Output table headings
Output HP, VP, QP, CP at each DX increment, N+1 points.

↓
A. Update time and commence unsteady flow simulation.

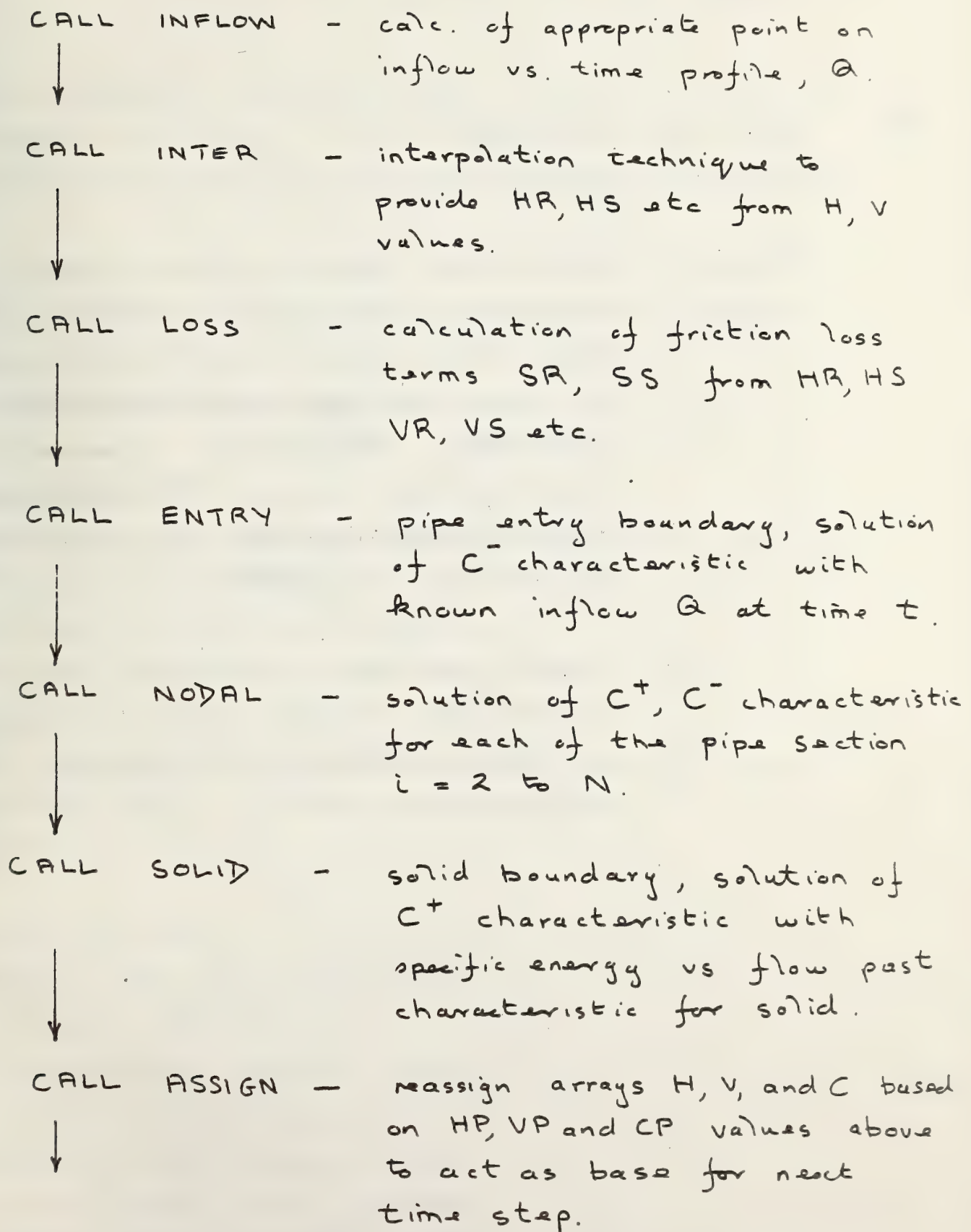
↓
 $TIME = TIME + DT.$

↓
CHECK TIME VS TMAX

↙
TIME < TMAX, GOTO B

↘
TIME ≥ TMAX, GOTO E

Note - unsteady flow simulation achieved via a series of subroutines as follows



WRITE - values of HP, VP, QP, CP for
each calculated point, $i = 1, N+1$.
↓
GOTO A

E. End of simulation

Note (i) Subroutine SHAPE called from most other subroutines. SHAPE calculates flow area, surface width etc based on circular pipe cross section geometry. Alternative channel shape investigations using TRANSCA only require replacement of geometry equations in SHAPE.

(ii) Double precision calculation required, mainly due to the need to $\sqrt{\text{differences}}$ of large numbers, DP. need if these differences small.

SAMPLE INPUT DATA PROGRAM TRANSCA.

Line 1

Pipe diameter, Manning coeff., slope, length.

Format 4F10.4

▽▽▽▽ 0.1000 ▽▽▽▽ 0.0150 ▽▽▽▽ 0.0033 ▽▽▽▽ 5.0000

Line 2

Solid minimum specific energy, SE vs Q coeff, XK.

Format 2F10.4

▽▽▽▽ 0.0200 ▽▽▽▽ 0.6000

Line 3

N° calculation sections, TMAX, Time step factor

Format I3, 2F10.4

▽10 ▽▽▽ 25.0000 ▽▽▽▽ 5.0000

Line 4

N° pairs of coordinates on inflow-time curve.

Format I3

▽▽ 5

Lines 5-9

Inflow QIN at time TIN

Format 2F10.4

▽▽▽▽ 0.2000 ▽▽▽▽ 0.0000

▽▽▽▽ 1.0000 ▽▽▽▽ 2.5000

▽▽▽▽ 1.0000 ▽▽▽▽ 4.5000

▽▽▽▽ 0.2000 ▽▽▽▽ 9.5000

▽▽▽▽ 0.2000 ▽▽▽ 25.0000

Input data - units:-

S.I. units used in program TRANSC except specific energy in m.

S.I. units used in data input fields except for inflow profile data. Q_{IN} is read in l/s and converted to m^3/s in the program. Q_P output is l/s , otherwise SI used in output fields

Note value of XK , coefficient in solid boundary equation $Q = XK (SE - SE\phi)^2$ is read in m^2/s .

00000000

TRANSCA IS A PROGRAM DESIGNED TO APPLY THE METHOD OF CHARACTERISTICS SOLUTION TO THE EQUATIONS DEFINING UNSTEADY FLOW IN PARTIALLY FILLED CHANNELS.

```

DOUBLE PRECISION DP(30),QIN(30),TIN(30),VP(15),HP(15),CP(15)
DOUBLE PRECISION V(15),H(15)
DOUBLE PRECISION VR(15),HR(15),CR(15),VS(15),HS(15),CS(15),SS(15)
DOUBLE PRECISION SR(15),XR(15),XS(15),XN(15),C(15)
DOUBLE PRECISION TIME, D,HP,SO,PL,SEQ,XK,TFAC,TFAC
DOUBLE PRECISION HN,HC,A,T,PER,CN,SLQ,Q
DOUBLE PRECISION DX,DT,DTG,DTAT
COMMON/CM1/DTG,DT,DX,D,SO,PL,SEQ,XK,XN
COMMON/CM2/N,THAX
COMMON/CM3/NPTS,QIN,TIN
COMMON/CM4/V,H,C,VR,HR,CR,XK,SR,VS,HS,CS,XS,SS,XN
COMMON/CM5/HC,HN
COMMON/CM6/QP,VP,HP,CP

```

SET INITIAL CONDITIONS

TIME=0.0

READ PIPE DESCRIPTION DATA

READ(4,100)D,HP,SO,PL

D=DIA. RM-MANNING COEFF. SO=PIPE SLOPE. PL=PIPE LENGTH

FORMAT(4F10.4)

READ SOLID DESCRIPTION DATA

READ(4,101)SEQ,XK

SEQ-SPECIFIC ENERGY REQUIRED FOR FLOW PAST SOLID.

XK=FLOW TO SPECIFIC ENERGY COEFF. AT SP VALUES ABOVE SEQ.

FORMAT(2F10.4)

READ CALCULATION DATA

READ(4,103)N,THAX,TFAC

N=NUMBER OF PIPE SECTIONS CONSIDERED, THAX-DURATION OF CALC.

TFAC-TIME STEP FACTOR, 1-10.

FORMAT(15,2F10.4)

READ INFLOW DATA PROFILE. INFLOW PROFILE USED IS BASED ON A LINEAR INTERPOLATION BETWEEN THESE DATA POINTS. NOTE DATA READ IN IN L/S BUT USED IN PROGRAM IN M³/S.

READ(4,102)NPTS

FORMAT(13)

DO 50 I=1,NPTS

READ(4,104)QIN(I),TIN(I)

FORMAT(2F10.4)

QIN(I)=QIN(I)/1000.0

CONTINUE

CALCULATION OF STEADY UNIFORM FLOW DEPTH BASED ON INITIAL FLOW RATE QIN(1) PLUS CALCULATION OF INITIAL WAVESPEED AND LOSS

TERM SLO

CALL DEPTH(TIME)

CALL SHAPE(HN,A,T,PER)

CALL WAVSPD(HN,CN)

```

SLO=(RM**2)*((QIN(1)/A)**2)/(A/PL**2)**1.3333
ASSIGN TIME ZERO CONDITIONS ALONG TEST PIPE.
DO 51 I=1,N+1
  VP(I)=QIN(1)/A
  JP(I)=QIN(1)*1000.0
  HP(I)=HN
  CP(I)=CN
51 CONTINUE
  XN(I)=0.0
  DO 52 I=2,N+1
    SR(I)=SLO
    VR(I)=QIN(1)/A
    HR(I)=HN
    CR(I)=CN
    XN(I)=XN(I-1)+PL/FLQAT(N)
52 CONTINUE
  DO 53 I=1,N
    SS(I)=SLO
    VS(I)=QIN(1)/A
    HS(I)=HN
    CS(I)=CN
53 CONTINUE
  CALL ASSIGN(N)
  CALCULATION OF TIME STEP AND LENGTH SECTION.
  DX=PL/FLQAT(N)
  DT=DX/(17FAC*CN)
  DT=DT

  OUTPUT TEST DESCRIPTION PLUS INITIAL CONDITIONS.
  WRITE(3,202)D,RP,S0,PL
202 FORMAT(10H,/,10X,'TEST PIPE CONFIGURATION:-',
1 /10X,'DIAMETER = ',F10.4,' F.',
1 10X,'MANNING COEFFICIENT:-',F10.4,
1 /10X,'PIPE SLOPE = ',F10.4,' PIPE LENGTH = ',
1 F10.4,' M.',//)
  IF(HC.GT.HN) WRITE(3,200)HN,HL
  IF(HC.LE.HN) WRITE(3,201)HN,HL
200 FORMAT(10X,'FLOW SUPERCRITICAL, ', 'NORMAL DEPTH = ',F10.4,
1 ' M.', ' AND CRITICAL DEPTH = ',F10.4,' M.',//)
201 FORMAT(10X,'FLOW SUBCRITICAL, ', 'NORMAL DEPTH = ',
1 F10.4,' M. AND CRITICAL DEPTH = ',F10.4,' M.',//)
  WRITE(3,203)QIN(1),CN
203 FORMAT(10X,'INITIAL FLOW RATE = ',F10.4,' M**3/S.',
1 ' INITIAL DEPTH = ',F10.4,' M.',
2 ' INITIAL WAVE SPEED = ',F10.4,' M/S.',//)
  N1=N+1
  WRITE(3,204)DT,DX,N1
204 FORMAT(10X,'CALCULATION TIME STEP = ',F10.4,' S.',
1 ' LENGTH INCREMENT = ',F10.4,' F.', ' NUMBER OF MODES = ',13,//)
  OUTPUT TEST SIMULATION RESULTS.
  WRITE(3,205)(XN(I),I=1,N1)
205 FORMAT(10X,' POSITION FROM ',11F7.4,/,10X,' ENTRY M.',//)
  WRITE(3,206)TIME,(VP(I),I=1,N1)
206 FORMAT(2X,' TIME = ',F5.3,' S.', ' DEPTH',7X,' M=',11F7.4)
  WRITE(3,207)(VP(I),I=1,N1)
  WRITE(3,307)(CP(I),I=1,N1)
307 FORMAT(10X,' FLOW RATE L/S=',11F7.4)
  WRITE(3,208)(CP(I),I=1,N1)
207 FORMAT(10X,' VELOCITY M/S=',11F7.4)

```



```

SUBROUTINE DEPTH(TIME)
THIS SUBROUTINE USES A SECTION OF ENTRY TO CALCULATE NORMAL
AND CRITICAL FLOW DEPTHS.
DOUBLE PRECISION QIN(30),TIN(30)
DOUBLE PRECISION HC,HN,TIME
COMMON/CM3/MPTS,WIN,TIN
COMMON/CM5/HC,HN
CALL ENTRY(TIME)
RETURN
END

```

```

SUBROUTINE INFLOW(TIME,QAV)
THIS SUBROUTINE CALCULATES INFLOW RATES AT PIPE ENTRY BASED
ON THE ENTRY FLOW PROFILE DATA. NOTE THAT THE Q CALCULATED
IS AN AVERAGE VALUE FOR THIS TIME STEP.
DOUBLE PRECISION Q(30),T(30),QX(30)
DOUBLE PRECISION JAV,DT0,DT,DX,SO,PL,SEC,XK,KM
DOUBLE PRECISION T1,TJ,TX
COMMON/CM1/DT0,DT,DX,U,SO,PL,SEC,XK,KM
COMMON/CM3/MPTS,M,T
T1=TIME
TJ=TIME-DT
J=1
TX=T1
DO 3 I=1,MPTS-1
IF(TX.GE.T(I).AND.TX.LT.T(I+1)) GOTO 4
CONTINUE
QX(J)=Q(I)+(T(I+1)-TX)*(Q(I+1)-Q(I))/(T(I+1)-T(I))
JAV=QX(J)
RETURN
END

```

```

SUBROUTINE SHAPE(H,A,T,PER)
DOUBLE PRECISION DT0,DT,DX,U,SO,PL,SEC,XK,KM,PI
DOUBLE PRECISION H,A,T,PER,THETA
COMMON/CM1/DT0,DT,DX,U,SO,PL,SEC,XK,KM
THIS SUBROUTINE CALCULATES FLOW AREA, SURFACE WIDTH AND
WETTED PERIMETER BASED ON FLOW DEPTH
R=D/2.0
PI=3.142
IF(H.LT.R)THETA=2.0*DATAN(DSQRT(H*(D-H)))/(R-H)
IF(H.EQ.R)THETA=PI
IF(H.GT.R)THETA=PI+2.0*DATAN((H-R)/(DSQRT(H*(D-H))))
A=((D**2)/8.0)*(THETA-DSIN(THETA))
PER=D*THETA/2.0
T=2.0*((H*(D-H))**.5)
RETURN
END

```



```

SUBROUTINE WAVSPD(H,C)
DOUBLE PRECISION H,C,AREA,T,PEF
THIS SUBROUTINE CALCULATES WAVE SPEED BASED ON DEPTH AND CROSS
SECTION SHAPE.
CALL SHAPE(H,AREA,T,PEF)
C=DSQRT(9.81*AREA/T)
RETURN
END

```

```

SUBROUTINE INTER(N)
DOUBLE PRECISION V(15),H(15),C(15),VR(15),HR(15),CR(15),VS(15)
DOUBLE PRECISION HS(15),CS(15),SK(15),SS(15),AN(15)
DOUBLE PRECISION XS(15),XF(15)
DOUBLE PRECISION DT,DT0,CX,D,SO,PL,SLO,XK,RM
DOUBLE PRECISION THETA
COMMON/CM1/DT0,LT,XK,D,SO,PL,SEC,XK,RM
COMMON/CM4/V,H,C,VR,HR,CR,XK,SE,VS,HS,CS,AS,SS,XN
THIS SUBROUTINE SETS UP, BY INTERPOLATION, THE BASE CONDITIONS
FOR THE NEXT TIME STEP.
THETA=DT/CX
N1=N+1
DO 1 I=2,N1
VR(I)=(V(I)+THETA*(C(I)*V(I-1)-V(I)*C(I-1)))
1 / (1.0+THETA*(V(I)-V(I-1)+C(I)-C(I-1)))
CR(I)=(C(I)*(1.0-VR(I)*THETA)+C(I-1)*VR(I)*THETA)
1 / (1.0+C(I)*THETA-C(I-1)*THETA)
HR(I)=H(I)-(H(I)-H(I-1))*THETA*(VR(I)+CR(I))
XR(I)=XN(I)-(VR(I)+C(I))*DT
CONTINUE
DO 2 I=1,N
VS(I)=(V(I)-THETA*(V(I)*C(I+1)-C(I)*V(I+1)))
1 / (1.0-THETA*(V(I)-V(I+1)-C(I)+C(I+1)))
CS(I)=(C(I)+VS(I)*THETA*(C(I)-C(I+1)))
1 / (1.0+THETA*(C(I)-C(I+1)))
HS(I)=H(I)+THETA*(VS(I)-CS(I))*(H(I)-H(I+1))
XS(I)=XN(I)+(VS(I)-CS(I))*LT
CONTINUE
RETURN
END

```

```

SUBROUTINE ENTRY(TIME)
THIS SUBROUTINE CALCULATES THE UPSTREAM BOUNDARY CONDITIONS
AT EACH TIME STEP BASED ON A KNOWN INFLOW PROFILE.
DOUBLE PRECISION QP(30),QIN(30),TIN(30),VP(15),HP(15),CP(15)
DOUBLE PRECISION V(15),H(15)
DOUBLE PRECISION VR(15),HR(15),CR(15),VS(15),HS(15),CS(15),SS(15)

```

```

DOUBLE PRECISION SR(15),XF(15),XS(15),XM(15),C(15)
DOUBLE PRECISION TIME
DOUBLE PRECISION DT,DT,DX,D,SO,PL,SEO,XX,RM
DOUBLE PRECISION TMAX
DOUBLE PRECISION HC,HN
DOUBLE PRECISION G,CJN,D,UP,DN,AREA,T,PER,HCRIT,HCM
DOUBLE PRECISION MNORM,MNN,HE,X3,X4,HFLOW,HBS
COMMON/CM1/(TO,DT,DX,D,SO,PL,SEO,XX,RM
COMMON/CM2/(N,TMAX
COMMON/CM3/(FTS,QIN,TIN
COMMON/CM4/(V,H,C,VR,HR,CR,XR,SF,VS,H3,CS,XS,SS,XR
COMMON/CM5/(HC,HN
COMMON/CM6/(CF,VP,HP,CP
G=9.81
CON=RM**2/SC
IF (TIME.GT.0.0) CALL INFLOW(TIME,C)
IF (TIME.GT.0.0) GOTO 600
IF (TIME.EQ.0.0) G=QIN(1)
CALCULATION OF CRITICAL DEPTH.
JP=0
DN=0.0
HC=(UP+DN)/2.0
CONTINUE
CALL SHAPE(HC,AREA,T,PER)
HCRIT=1.0-(C**2)*T/(3*AREA**3)
IF (HCRIT)3,4,5
3 DN=HC
GOTO 6
5 UP=HC
HN=(UP+DN)/2.0
IF (ABS((HCN-HC)/HC).LE.0.001) GOTO 5
HC=HCN
GOTO 7
7 HC=HCN
CONTINUE
CALCULATION OF NORMAL DEPTH.
JP=0
DN=0.0
HN=(UP+DN)/2.0
CONTINUE
CALL SHAPE(HN,AREA,T,PER)
MNORM=1.0-(C**2)*QIN/((AREA**3.333)/(PER**1.333))
IF (MNN)10,11,12
10 DN=MN
GOTO 13
12 UP=MN
13 HNN=(UP+DN)/2.0
IF (ABS((HNN-HN)/HN).LE.0.001) GOTO 14
HN=HNN
GOTO 9
14 HN=HNN
CONTINUE
IF (TIME.EQ.0.0) GOTO 500
CALCULATION OF BOUNDARY DEPTH.
CONTINUE
JP=0
DN=0.0
HB=(UP+DN)/2.0
CONTINUE
CALL SHAPE(HB,AREA,T,PER)

```



```

SUBROUTINE SOLID(TIME)
DOUBLE PRECISION DP(30),QIN(30),TIN(30),VP(15),HP(15),CP(15)
DOUBLE PRECISION V(15),H(15)
DOUBLE PRECISION VR(15),HR(15),CR(15),VS(15),HS(15),CS(15),SS(15)
DOUBLE PRECISION FS(2),SR(15),XF(15),XS(15),XN(15),C(15)
DOUBLE PRECISION DT0,DT,DX,D,SO,FL,SEO,XK,RM,TIME
DOUBLE PRECISION TMAX,HC,HN,G,SEV,H0,A1,T,PER,X1
DOUBLE PRECISION X2,SE,U,E,H,Y,Z,Z4,Z3,Z2,Z1,Z0,F
DOUBLE PRECISION DF,H,A1,A2,SE2
COMMON/CH1/DT0,DT,DX,D,SO,PL,SEO,XK,XM
COMMON/CH2/N,TMAX
COMMON/CH3/NPTS,QIN,TIN
COMMON/CH4/V,H,C,VR,HR,CR,XK,SP,VS,HS,CS,XS,SS,XN
COMMON/CH5/HC,HN
COMMON/CH6/GF,VP,HP,CP
THIS SUBROUTINE CALCULATES THE FLOW DEPTH, LEAKAGE RATE AND
WAVE SPEED AT THE DOWNSTREAM BOUNDARY FORMED BY AN INITIALLY
STATIONARY SOLID.
G=9.81
IZ=0
SEV=SEO
J=1
H0=H(N+1)
CONTINUE
J=1
3000 CALL SHAPE(H0,A1,T,PER)
X1=G/CP(N+1)
X2=X1*HR(N+1)+VR(N+1)-G*(SR(N+1)-SO)*DT
SE=H(N+1)+(V(N+1)**2)/(2.0*C)
IF(SE.LT.SEO) GOTO 5
75 CONTINUE
U=X2*A1/XK
S=X1*A1/XK
W=(X1**2)/(2.0*G)
Y=(X2**2)/(2.0*G)-SEJ
Z=(1.0-X2*X1/G)
Z4=W**2
Z3=2.0*W*Z
Z2=Z**2+2.0*W*Y
Z1=2.0*Y*Z+E
Z0=Y**2-U
F=Z4*H0**4+Z3*H0**3+Z2*H0**2+Z1*H0+Z0
FS(J)=F
IF(J.EQ.2) GOTO 7000
J=2
H0=H0+0.0001*H0
GOTO 8000
7000 DF=(FS(2)-FS(1))/(40/10000.0)
DH=FS(1)/DF
H1=H0-DH
IF(ABS((H1-H0)/H0).LE.0.005) GOTO 70
IF(H1.GT.0) H1=1.1*H0
H0=H1
IZ=IZ+1
IF(IZ.GT.60) TIME=TMAX
IF(IZ.GT.60) GOTO 301
IF(IZ.GT.10) GOTO 805
GOTO 60
70 CALL SHAPE(H1,A2,T,PER)
IF(ABS((A2-A1)/A1).LE.0.005) GOTO 80
A1=(A1+A2)/2.0
H0=H1
GOTO 65

```



```

30    CONTINUE
      HP(N+1)=H1
      VP(N+1)=X2-X1*H1
      JP(N+1)=VF(N+1)*A2*1000.0
      GOTO 5
6     VP(N+1)=0.0
      HP(N+1)=X2/X1
      JP(N+1)=0.0
      GOTO 5
5     CALL WAVSFD(HP(N+1),CP(N+1))
      SE2=HP(N+1)+(VP(N+1)**2)/(2.0*C)
      IF(SE2.GT.SE0.AND.SE.LT.SE0)GOTO 800
      GOTO 801
300   SE0=SE
      H0=H(N+1)
      GOTO 65
301   CONTINUE
      SE0=SEV
      DT=DT0
      GOTO 806
305   CONTINUE
      TIME=TIME-DT
      DT=DT/2.0
      GOTO 806
306   CONTINUE
      RETURN
      END

SUBROUTINE ASSIGN(N)
  THIS SUBROUTINE SETS UP THE NEW BASE CONDITIONS ALONG THE
  PIPE IN PREPARATION FOR THE NEXT TIME STEP.
  DOUBLE PRECISION QP(30),QIN(30),TIN(30),VP(15),HP(15),CP(15)
  DOUBLE PRECISION V(15),H(15)
  DOUBLE PRECISION VR(15),HR(15),CR(15),VS(15),HS(15),CS(15),S,(15)
  DOUBLE PRECISION SR(15),XF(15),XS(15),XN(15),C(15)
  COMMON/CM3/NPTS,QIN,TIN
  COMMON/CM4/V,H,C,VR,HR,CR,XF,SF,VS,HS,CS,XS,SS,XN
  COMMON/CM6/QP,VP,HP,CP
  DO 1 I=1,N+1
    V(I)=VP(I)
    H(I)=HP(I)
    C(I)=CP(I)
1    CONTINUE
    RETURN
    END
$BEND

```

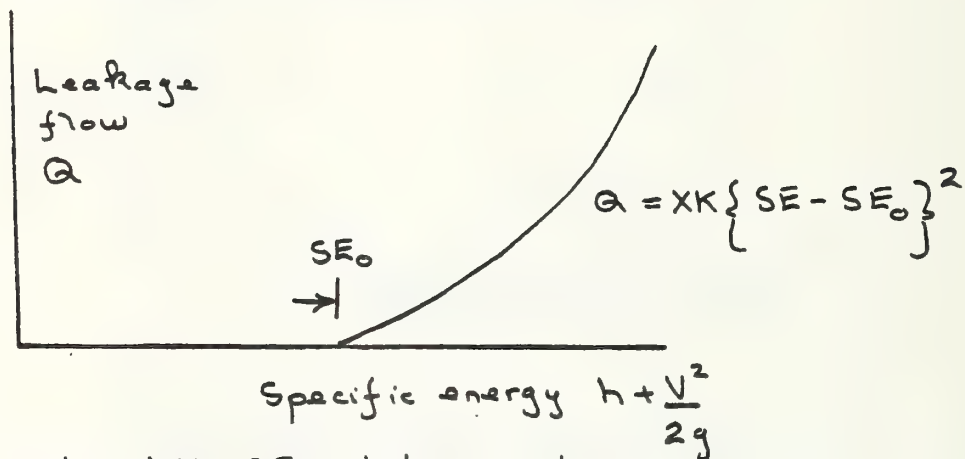
Appendix 2

Derivation of solid boundary equations
used in Subroutine SOLID

Derivation of solid boundary equations used in Subroutine SOLID.

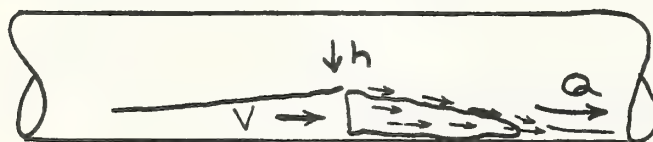
Available equations at solid boundary.

1) Leakage flow past solid:-



Note XK , SE_0 determined by measurement.

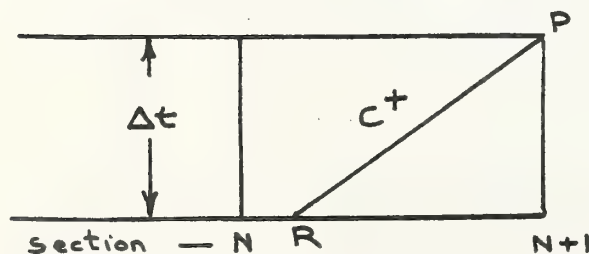
2) Continuity of flow over solid.



$$VA = Q$$

$$A = f(h)$$

3) Characteristic equation, C^+ linking R at t to P at $t + \Delta t$.



C^+ equation of form

$$V_{N+1} = X_2 - X_1 h_{N+1}$$

Values of $X_1 = g/c_R$

$$X_2 = V_R + gh_R/c_R - g(S_R - S_0)\Delta t$$

From available equations at solid boundary,

$$Q = VA = (X_2 - X_1 h)A = XK \{ SE - SE_0 \}^2$$

$$= XK \left\{ h + \frac{V^2}{2g} - SE_0 \right\}^2$$

$$(X_2 - X_1 h) \frac{A}{XK} = \left\{ h + \frac{(X_2 - X_1 h)^2}{2g} - SE_0 \right\}^2$$

$$= \left\{ h + \frac{(X_2^2 + X_1^2 h^2 - 2X_2 X_1 h)}{2g} - SE_0 \right\}^2$$

Collecting terms

$$U = X_2 \cdot A / XK ; \quad B = X_1 \cdot A / XK ; \quad Y = X_2^2 / 2g - SE_0 ;$$

$$W = X_1^2 / 2g ; \quad Z = 1 - 2 \cdot X_1 \cdot X_2 / 2g$$

$$\therefore U - Bh = \left\{ Zh + Y + Wh^2 \right\}^2$$

$$= Z^2 h^2 + (Y + Wh^2)^2 + 2Zh(Y + Wh^2)$$

$$= Z^2 h^2 + Y^2 + W^2 h^4 + 2YWh^2 + 2ZYh + 2ZWh^3$$

$$W^2 h^4 + 2ZWh^3 + (Z^2 + 2YW)h^2 + (2ZY + B)h + Y^2 - U = 0$$

The quartic in h cannot be solved directly
as $A = f(h)$ and therefore B and $U = f(h)$

Solution by Newton Raphson method is
acceptable and employed in Subroutine SOLID.

Appendix 3
TRANSCA output

TEST PIPE COMPLETION:-
 DIAMETER = 0.1000 M.
 PIPE SLOPE = 0.0012 FLOWS COLLECTED = 0.0100 M³

FLOW SURFICIAL, NORMAL VELOCITY = 0.0109 M. AND CRITICAL DEPTH = 0.0116 M.

INITIAL FLOW RATE = 0.0062 M³/S. INITIAL DEPTH = 0.0189 M INITIAL WAVE SLOPE = 0.3500 F/S.
 CALCULATION TIME STEP = 0.0700 S. LENGTH INCREMENT = 0.5000 M. NUMBER OF MODES = 11

POSITION FROM 0. 0.5000 1.0000 1.5000 2.0000 2.5000 3.0000 3.5000 4.0000 4.5000 5.0000

TIME = 0. S. DEPTH M= 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189
 VELOCITY M/S= 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945
 FLOW RATE L/S= 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000
 WAVE SPD. M/S= 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590

TIME = 0.279 S. DEPTH M= 0.0207 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189
 VELOCITY M/S= 0.2459 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946
 FLOW RATE L/S= 0.2896 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000
 WAVE SPD. M/S= 0.3775 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590

TIME = 0.557 S. DEPTH M= 0.0224 0.0194 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189
 VELOCITY M/S= 0.2668 0.2104 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946
 FLOW RATE L/S= 0.3181 0.2260 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000
 WAVE SPD. M/S= 0.3937 0.3684 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590

TIME = 0.836 S. DEPTH M= 0.0241 0.0204 0.0190 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189
 VELOCITY M/S= 0.3211 0.2351 0.1993 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946
 FLOW RATE L/S= 0.4477 0.2704 0.2077 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000
 WAVE SPD. M/S= 0.4086 0.3782 0.3608 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590

TIME = 1.114 S. DEPTH M= 0.0257 0.0216 0.0194 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189
 VELOCITY M/S= 0.3561 0.2683 0.2103 0.1960 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946
 FLOW RATE L/S= 0.5177 0.3102 0.2259 0.2023 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000
 WAVE SPD. M/S= 0.4229 0.3858 0.3649 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590

TIME = 1.393 S. DEPTH M= 0.0272 0.0230 0.0201 0.0191 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189
 VELOCITY M/S= 0.3746 0.2946 0.2273 0.2033 0.1950 0.1946 0.1946 0.1946 0.1946 0.1946 0.1946
 FLOW RATE L/S= 0.6467 0.4023 0.2564 0.2022 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000 0.2000
 WAVE SPD. M/S= 0.4383 0.3950 0.3715 0.3611 0.3592 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590

TIME = 1.671 S. DEPTH M= 0.0286 0.0245 0.0210 0.0194 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189
 VELOCITY M/S= 0.3956 0.3243 0.2495 0.2054 0.1965 0.1947 0.1947 0.1947 0.1947 0.1947 0.1947
 FLOW RATE L/S= 0.7346 0.4630 0.2898 0.2229 0.2012 0.2002 0.2000 0.2000 0.2000 0.2000 0.2000
 WAVE SPD. M/S= 0.4490 0.4127 0.3803 0.3663 0.3591 0.3590 0.3590 0.3590 0.3590 0.3590 0.3590

TIME = 1.950 S. DEPTH M= 0.0301 0.0259 0.0222 0.0199 0.0191 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189
 VELOCITY M/S= 0.4144 0.3514 0.2751 0.2211 0.2001 0.1952 0.1947 0.1947 0.1947 0.1947 0.1947
 FLOW RATE L/S= 0.8222 0.5091 0.3263 0.2453 0.2004 0.2011 0.2000 0.2000 0.2000 0.2000 0.2000
 WAVE SPD. M/S= 0.4614 0.4257 0.3911 0.3692 0.3611 0.3593 0.3591 0.3590 0.3590 0.3590 0.3590

TIME = 2.220 S. DEPTH M= 0.0315 0.0275 0.0235 0.0206 0.0193 0.0189 0.0189 0.0189 0.0189 0.0189 0.0189
 VELOCITY M/S= 0.4307 0.3759 0.3023 0.2353 0.2064 0.1967 0.1947 0.1947 0.1947 0.1947 0.1947
 FLOW RATE L/S= 0.9115 0.5591 0.3684 0.2779 0.2014 0.2014 0.2000 0.2000 0.2000 0.2000 0.2000
 WAVE SPD. M/S= 0.4733 0.4390 0.4031 0.3761 0.3615 0.3593 0.3592 0.3590 0.3590 0.3590 0.3590

TIME = 2.507 S. DEPTH M= 0.0329 0.0290 0.0249 0.0215 0.0197 0.0190 0.0189 0.0189 0.0189 0.0189 0.0189

TIME = 13.091 S.	M= 0.0224	0.0230	0.0237	0.0247	0.0261	0.0265	0.0317	0.3324	0.0385	0.0412	0.0431
DEPTH	M/S= 0.1521	0.1580	0.1587	0.1674	0.1633	0.1590	0.1376	0.1147	0.1013	0.0992	0.1035
VELOCITY	L/S= 0.2103	0.2153	0.2142	0.2297	0.2297	0.2297	0.2297	0.2297	0.2297	0.2297	0.2297
FLOW RATE	M/S= 0.3930	0.3986	0.4051	0.4139	0.4271	0.4474	0.4649	0.4790	0.5112	0.5514	0.5664
WAVE SPD.	M= 0.0223	0.0229	0.0236	0.0246	0.0261	0.0285	0.0318	0.0354	0.0385	0.0410	0.0428
DEPTH	M/S= 0.1535	0.1595	0.1630	0.1677	0.1672	0.1696	0.1252	0.1031	0.0947	0.0957	0.1019
VELOCITY	L/S= 0.2102	0.2147	0.2105	0.2292	0.2255	0.2275	0.2610	0.2646	0.2696	0.2726	0.2776
FLOW RATE	M/S= 0.3921	0.3977	0.4042	0.4131	0.4267	0.4475	0.4649	0.5109	0.5301	0.5495	0.5644
WAVE SPD.	M= 0.0222	0.0228	0.0235	0.0245	0.0261	0.0285	0.0318	0.0353	0.0383	0.0407	0.0426
DEPTH	M/S= 0.1541	0.1598	0.1623	0.1631	0.1662	0.1606	0.1161	0.0963	0.0891	0.0922	0.1003
VELOCITY	L/S= 0.1998	0.2140	0.2285	0.2434	0.2560	0.2597	0.2497	0.2340	0.2469	0.2723	0.3155
FLOW RATE	M/S= 0.3912	0.3969	0.4035	0.4125	0.4265	0.4480	0.4760	0.5096	0.5238	0.5475	0.5623
WAVE SPD.	M= 0.0221	0.0227	0.0234	0.0245	0.0261	0.0286	0.0319	0.0353	0.0382	0.0404	0.0423
DEPTH	M/S= 0.1552	0.1591	0.1617	0.1606	0.1517	0.1319	0.1065	0.0864	0.0638	0.0891	0.0985
VELOCITY	L/S= 0.2062	0.2132	0.2266	0.2390	0.2472	0.2497	0.2297	0.2140	0.2307	0.2653	0.3111
FLOW RATE	M/S= 0.3905	0.3961	0.4026	0.4122	0.4267	0.4487	0.4766	0.5043	0.5273	0.5454	0.5600
WAVE SPD.	M= 0.0220	0.0226	0.0234	0.0244	0.0261	0.0287	0.0320	0.0352	0.0379	0.0401	0.0420
DEPTH	M/S= 0.1562	0.1594	0.1610	0.1500	0.1462	0.1236	0.0976	0.0815	0.0792	0.0862	0.0967
VELOCITY	L/S= 0.2005	0.2126	0.2248	0.2349	0.2370	0.2306	0.2115	0.2014	0.2165	0.2541	0.3025
FLOW RATE	M/S= 0.3898	0.3954	0.4023	0.4120	0.4272	0.4498	0.4774	0.5038	0.5257	0.5431	0.5577
WAVE SPD.	M= 0.0219	0.0226	0.0233	0.0244	0.0261	0.0289	0.0321	0.0352	0.0377	0.0399	0.0417
DEPTH	M/S= 0.1564	0.1596	0.1602	0.1553	0.1497	0.1156	0.0865	0.0734	0.0752	0.0835	0.0948
VELOCITY	L/S= 0.1995	0.2120	0.2230	0.2310	0.2311	0.2173	0.1950	0.1829	0.2039	0.2437	0.2937
FLOW RATE	M/S= 0.3889	0.3946	0.4016	0.4120	0.4271	0.4491	0.4716	0.5013	0.5240	0.5408	0.5552
WAVE SPD.	M= 0.0219	0.0225	0.0233	0.0245	0.0263	0.0290	0.0322	0.0351	0.0375	0.0396	0.0414
DEPTH	M/S= 0.1576	0.1597	0.1593	0.1526	0.1352	0.1081	0.0825	0.0703	0.0717	0.0810	0.0929
VELOCITY	L/S= 0.2005	0.2111	0.2213	0.2271	0.2271	0.2047	0.1802	0.1727	0.1929	0.2341	0.2850
FLOW RATE	M/S= 0.3884	0.3941	0.4015	0.4122	0.4233	0.4454	0.4715	0.5019	0.5205	0.5362	0.5503
WAVE SPD.	M= 0.0218	0.0224	0.0233	0.0245	0.0265	0.0292	0.0323	0.0350	0.0373	0.0393	0.0411
DEPTH	M/S= 0.1581	0.1598	0.1582	0.1496	0.1297	0.1010	0.0763	0.0659	0.0687	0.0787	0.0910
VELOCITY	L/S= 0.2002	0.2106	0.2194	0.2233	0.2172	0.1929	0.1672	0.1616	0.1835	0.2253	0.2765
FLOW RATE	M/S= 0.3877	0.3936	0.4012	0.4126	0.4331	0.4539	0.4775	0.5019	0.5205	0.5362	0.5503
WAVE SPD.	M= 0.0217	0.0224	0.0232	0.0246	0.0266	0.0294	0.0323	0.0349	0.0371	0.0390	0.0407
DEPTH	M/S= 0.1585	0.1594	0.1571	0.1464	0.1261	0.0944	0.0710	0.0624	0.0663	0.0767	0.0892
VELOCITY	L/S= 0.1997	0.2099	0.2176	0.2173	0.2082	0.1819	0.1560	0.1524	0.1756	0.2175	0.2682
FLOW RATE	M/S= 0.3870	0.3932	0.4011	0.4131	0.4314	0.4554	0.4801	0.5012	0.5187	0.5339	0.5479
WAVE SPD.	M= 0.0217	0.0223	0.0232	0.0246	0.0268	0.0296	0.0324	0.0348	0.0369	0.0387	0.0404
DEPTH	M/S= 0.1563	0.1586	0.1557	0.1430	0.1176	0.0884	0.0665	0.0576	0.0643	0.0750	0.0874
VELOCITY	L/S= 0.2000	0.2090	0.2157	0.2150	0.2037	0.1717	0.1475	0.1420	0.1615	0.2035	0.2563
FLOW RATE	M/S= 0.3866	0.3927	0.4011	0.4136	0.4310	0.4570	0.4805	0.5004	0.5172	0.5317	0.5455
WAVE SPD.	M= 0.0216	0.0223	0.0233	0.0247	0.0270	0.0298	0.0324	0.0347	0.0367	0.0384	0.0402
DEPTH	M/S= 0.1565	0.1594	0.1541	0.1394	0.1130	0.0829	0.0629	0.0575	0.0629	0.0735	0.0857
VELOCITY	L/S= 0.2002	0.2093	0.2137	0.2110	0.1912	0.1624	0.1316	0.1142	0.1618	0.2053	0.2527
FLOW RATE	M/S= 0.3861	0.3924	0.4012	0.4147	0.4316	0.4504	0.4691	0.4885	0.5123	0.5296	0.5432
WAVE SPD.	M= 0.0216	0.0223	0.0233	0.0249	0.0272	0.0299	0.0325	0.0346	0.0365	0.0382	0.0399
DEPTH	M/S= 0.1603	0.1591	0.1524	0.1336	0.1075	0.0780	0.0579	0.0523	0.0617	0.0722	0.0840
VELOCITY	L/S= 0.2000	0.2075	0.2116	0.2066	0.1893	0.1540	0.1347	0.1149	0.1547	0.1989	0.2484
FLOW RATE	M/S= 0.3856	0.3921	0.4014	0.4136	0.4331	0.4531	0.4717	0.4897	0.5117	0.5275	0.5410
WAVE SPD.	M= 0.0215	0.0223	0.0233	0.0250	0.0274	0.0298	0.0321	0.0345	0.0363	0.0379	0.0394
DEPTH	M/S= 0.1600	0.1590	0.1504	0.1316	0.1074	0.0770	0.0576	0.0549	0.0641	0.0741	0.0874
VELOCITY	L/S= 0.1997	0.2067	0.2094	0.2023	0.1776	0.1466	0.1273	0.1118	0.1562	0.1943	0.2317
FLOW RATE	M/S= 0.3850	0.3907	0.4004	0.4126	0.4310	0.4491	0.4675	0.4855	0.5035	0.5215	0.5395
WAVE SPD.	M= 0.0215	0.0221	0.0231	0.0249	0.0274	0.0298	0.0321	0.0345	0.0363	0.0379	0.0394
DEPTH	M/S= 0.1600	0.1590	0.1504	0.1316	0.1074	0.0770	0.0576	0.0549	0.0641	0.0741	0.0874
VELOCITY	L/S= 0.1997	0.2067	0.2094	0.2023	0.1776	0.1466	0.1273	0.1118	0.1562	0.1943	0.2317
FLOW RATE	M/S= 0.3850	0.3907	0.4004	0.4126	0.4310	0.4491	0.4675	0.4855	0.5035	0.5215	0.5395
WAVE SPD.	M= 0.0215	0.0221	0.0231	0.0249	0.0274	0.0298	0.0321	0.0345	0.0363	0.0379	0.0394

TIME = 16.712 S.	WAVE SPD.	M/S = 0.1852	0.4920	0.4018	0.4169	0.4330	0.4611	0.4903	0.4978	0.5121	0.5250	0.5389
	DEPTH	M = 0.0215	0.0223	0.0234	0.0251	0.0276	0.0302	0.0325	0.0349	0.0361	0.0377	0.0394
	VELOCITY	M/S = 0.1812	0.1550	0.1403	0.1275	0.0974	0.0701	0.0559	0.0543	0.0604	0.0702	0.0610
	FLOW RATE	L/S = 0.2002	0.2057	0.2070	0.1972	0.1715	0.1403	0.1237	0.1208	0.1543	0.1904	0.2326
	WAVE SPD.	M/S = 0.3040	0.3919	0.4023	0.4103	0.4317	0.4622	0.4913	0.4969	0.5136	0.5237	0.5369
TIME = 16.970 S.	WAVE SPD.	M = 0.0215	0.0223	0.0234	0.0251	0.0276	0.0302	0.0325	0.0349	0.0361	0.0377	0.0394
	DEPTH	M/S = 0.1812	0.1550	0.1403	0.1275	0.0974	0.0701	0.0559	0.0543	0.0604	0.0702	0.0610
	VELOCITY	M/S = 0.2002	0.2057	0.2070	0.1972	0.1715	0.1403	0.1237	0.1208	0.1543	0.1904	0.2326
	FLOW RATE	L/S = 0.3040	0.3919	0.4023	0.4103	0.4317	0.4622	0.4913	0.4969	0.5136	0.5237	0.5369
	WAVE SPD.	M/S = 0.3040	0.3919	0.4023	0.4103	0.4317	0.4622	0.4913	0.4969	0.5136	0.5237	0.5369
TIME = 17.269 S.	WAVE SPD.	M = 0.0215	0.0223	0.0234	0.0251	0.0276	0.0302	0.0325	0.0349	0.0361	0.0377	0.0394
	DEPTH	M/S = 0.1812	0.1550	0.1403	0.1275	0.0974	0.0701	0.0559	0.0543	0.0604	0.0702	0.0610
	VELOCITY	M/S = 0.2002	0.2057	0.2070	0.1972	0.1715	0.1403	0.1237	0.1208	0.1543	0.1904	0.2326
	FLOW RATE	L/S = 0.3040	0.3919	0.4023	0.4103	0.4317	0.4622	0.4913	0.4969	0.5136	0.5237	0.5369
	WAVE SPD.	M/S = 0.3040	0.3919	0.4023	0.4103	0.4317	0.4622	0.4913	0.4969	0.5136	0.5237	0.5369
TIME = 17.547 S.	WAVE SPD.	M = 0.0214	0.0223	0.0236	0.0256	0.0281	0.0305	0.0324	0.0341	0.0356	0.0371	0.0387
	DEPTH	M/S = 0.1618	0.1555	0.1405	0.1146	0.0944	0.0627	0.0539	0.0547	0.0605	0.0687	0.0772
	VELOCITY	M/S = 0.1999	0.2030	0.1991	0.1823	0.1529	0.1271	0.1190	0.1200	0.1515	0.1620	0.2108
	FLOW RATE	L/S = 0.3043	0.3922	0.4045	0.4227	0.4445	0.4667	0.4808	0.4942	0.5065	0.5189	0.5318
	WAVE SPD.	M/S = 0.3043	0.3922	0.4045	0.4227	0.4445	0.4667	0.4808	0.4942	0.5065	0.5189	0.5318
TIME = 17.826 S.	WAVE SPD.	M = 0.0214	0.0224	0.0239	0.0260	0.0283	0.0305	0.0324	0.0339	0.0354	0.0369	0.0385
	DEPTH	M/S = 0.1621	0.1530	0.1345	0.1063	0.0771	0.0603	0.0545	0.0562	0.0615	0.0682	0.0751
	VELOCITY	M/S = 0.2002	0.2007	0.1932	0.1724	0.1433	0.1223	0.1179	0.1314	0.1523	0.1787	0.2082
	FLOW RATE	L/S = 0.3043	0.3924	0.4079	0.4285	0.4403	0.4658	0.4797	0.4917	0.5032	0.5151	0.5278
	WAVE SPD.	M/S = 0.3043	0.3924	0.4079	0.4285	0.4403	0.4658	0.4797	0.4917	0.5032	0.5151	0.5278
TIME = 18.383 S.	WAVE SPD.	M = 0.0214	0.0224	0.0240	0.0262	0.0285	0.0306	0.0321	0.0337	0.0352	0.0366	0.0382
	DEPTH	M/S = 0.1618	0.1515	0.1313	0.1024	0.0754	0.0596	0.0552	0.0571	0.0621	0.0682	0.0742
	VELOCITY	M/S = 0.1999	0.1995	0.1901	0.1677	0.1375	0.1219	0.1211	0.1332	0.1531	0.1726	0.2046
	FLOW RATE	L/S = 0.3043	0.3934	0.4079	0.4285	0.4403	0.4658	0.4797	0.4917	0.5032	0.5151	0.5278
	WAVE SPD.	M/S = 0.3043	0.3934	0.4079	0.4285	0.4403	0.4658	0.4797	0.4917	0.5032	0.5151	0.5278
TIME = 18.662 S.	WAVE SPD.	M = 0.0215	0.0225	0.0241	0.0264	0.0287	0.0306	0.0322	0.0336	0.0350	0.0365	0.0381
	DEPTH	M/S = 0.1618	0.1498	0.1260	0.0967	0.0733	0.0596	0.0551	0.0562	0.0629	0.0682	0.0734
	VELOCITY	M/S = 0.2002	0.1981	0.1876	0.1634	0.1364	0.1216	0.1224	0.1342	0.1541	0.1767	0.2015
	FLOW RATE	L/S = 0.3045	0.3941	0.4092	0.4291	0.4442	0.4659	0.4792	0.4909	0.5022	0.5140	0.5267
	WAVE SPD.	M/S = 0.3045	0.3941	0.4092	0.4291	0.4442	0.4659	0.4792	0.4909	0.5022	0.5140	0.5267
TIME = 19.040 S.	WAVE SPD.	M = 0.0215	0.0226	0.0243	0.0265	0.0283	0.0306	0.0322	0.0336	0.0349	0.0364	0.0379
	DEPTH	M/S = 0.1615	0.1491	0.1247	0.0954	0.0717	0.0593	0.0572	0.0594	0.0645	0.0682	0.0727
	VELOCITY	M/S = 0.2002	0.1967	0.1837	0.1591	0.1360	0.1220	0.1246	0.1374	0.1553	0.1761	0.1988
	FLOW RATE	L/S = 0.3046	0.3951	0.4105	0.4335	0.4450	0.4659	0.4791	0.4901	0.5013	0.5131	0.5256
	WAVE SPD.	M/S = 0.3046	0.3951	0.4105	0.4335	0.4450	0.4659	0.4791	0.4901	0.5013	0.5131	0.5256
TIME = 19.219 S.	WAVE SPD.	M = 0.0215	0.0227	0.0244	0.0267	0.0283	0.0306	0.0321	0.0335	0.0348	0.0363	0.0378
	DEPTH	M/S = 0.1608	0.1467	0.1214	0.0924	0.0705	0.0603	0.0594	0.0606	0.0644	0.0684	0.0721
	VELOCITY	M/S = 0.1997	0.1953	0.1805	0.1574	0.1323	0.1229	0.1271	0.1398	0.1567	0.1758	0.1962
	FLOW RATE	L/S = 0.3050	0.3957	0.4119	0.4317	0.4450	0.4659	0.4792	0.4904	0.5005	0.5122	0.5247
	WAVE SPD.	M/S = 0.3050	0.3957	0.4119	0.4317	0.4450	0.4659	0.4792	0.4904	0.5005	0.5122	0.5247
TIME = 19.497 S.	WAVE SPD.	M = 0.0216	0.0228	0.0246	0.0268	0.0284	0.0306	0.0320	0.0334	0.0347	0.0362	0.0377
	DEPTH	M/S = 0.1604	0.1461	0.1203	0.0919	0.0697	0.0610	0.0607	0.0619	0.0656	0.0685	0.0715
	VELOCITY	M/S = 0.1994	0.1917	0.1774	0.1522	0.1266	0.1246	0.1266	0.1362	0.1561	0.1756	0.1940
	FLOW RATE	L/S = 0.3054	0.3996	0.4134	0.4332	0.4451	0.4657	0.4791	0.4907	0.5014	0.5114	0.5239
	WAVE SPD.	M/S = 0.3054	0.3996	0.4134	0.4332	0.4451	0.4657	0.4791	0.4907	0.5014	0.5114	0.5239
TIME = 19.776 S.	WAVE SPD.	M = 0.0216	0.0229	0.0248	0.0270	0.0289	0.0306	0.0320	0.0333	0.0347	0.0361	0.0376
	DEPTH	M/S = 0.1604	0.1449	0.1185	0.0896	0.0693	0.0619	0.0611	0.0632	0.0660	0.0687	0.0710
	VELOCITY	M/S = 0.1994	0.1872	0.1744	0.1494	0.1307	0.1259	0.1323	0.1486	0.1595	0.1755	0.1920
	FLOW RATE	L/S = 0.3059	0.3977	0.4140	0.4344	0.4451	0.4655	0.4772	0.4901	0.4991	0.5107	0.5232
	WAVE SPD.	M/S = 0.3059	0.3977	0.4140	0.4344	0.4451	0.4655	0.4772	0.4901	0.4991	0.5107	0.5232
TIME = 20.054 S.	WAVE SPD.	M = 0.0217	0.0230	0.0249	0.0271	0.0290	0.0306	0.0319	0.0332	0.0346	0.0360	0.0376
	DEPTH	M/S = 0.1604	0.1449	0.1185	0.0896	0.0693	0.0619	0.0611	0.0632	0.0660	0.0687	0.0710
	VELOCITY	M/S = 0.1994	0.1872	0.1744	0.1494	0.1307	0.1259	0.1323	0.1486	0.1595	0.1755	0.1920
	FLOW RATE	L/S = 0.3059	0.3977	0.4140	0.4344	0.4451	0.4655	0.4772	0.4901	0.4991	0.5107	0.5232
	WAVE SPD.	M/S = 0.3059	0.3977	0.4140	0.4344	0.4451	0.4655	0.4772	0.4901	0.4991	0.5107	0.5232

TIME = 20.333 S.	VELOCITY	M/S = 0.1193	0.1137	0.1112	0.0915	0.0692	0.0629	0.0645	0.0669	0.0686
	FLOW RATE	L/S = 0.0000	0.0195	0.0176	0.0193	0.0193	0.0191	0.0191	0.0191	0.0192
	WAVE SPD.	M/S = 0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TIME = 20.611 S.	VELOCITY	M/S = 0.0216	0.0211	0.0202	0.0202	0.0200	0.0200	0.0200	0.0200	0.0200
	FLOW RATE	L/S = 0.0000	0.0195	0.0190	0.0190	0.0190	0.0190	0.0190	0.0190	0.0190
	WAVE SPD.	M/S = 0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TIME = 20.890 S.	VELOCITY	M/S = 0.0219	0.0234	0.0254	0.0274	0.0290	0.0304	0.0317	0.0330	0.0344
	FLOW RATE	L/S = 0.0000	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195
	WAVE SPD.	M/S = 0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TIME = 21.168 S.	VELOCITY	M/S = 0.0221	0.0236	0.0255	0.0274	0.0290	0.0304	0.0317	0.0330	0.0343
	FLOW RATE	L/S = 0.0000	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195
	WAVE SPD.	M/S = 0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TIME = 21.447 S.	VELOCITY	M/S = 0.0224	0.0237	0.0256	0.0275	0.0290	0.0304	0.0316	0.0329	0.0343
	FLOW RATE	L/S = 0.0000	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195
	WAVE SPD.	M/S = 0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TIME = 21.725 S.	VELOCITY	M/S = 0.0223	0.0239	0.0258	0.0275	0.0290	0.0303	0.0316	0.0329	0.0342
	FLOW RATE	L/S = 0.0000	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195
	WAVE SPD.	M/S = 0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TIME = 22.004 S.	VELOCITY	M/S = 0.0224	0.0250	0.0259	0.0276	0.0290	0.0303	0.0315	0.0328	0.0342
	FLOW RATE	L/S = 0.0000	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195
	WAVE SPD.	M/S = 0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TIME = 22.282 S.	VELOCITY	M/S = 0.0225	0.0241	0.0260	0.0276	0.0290	0.0302	0.0315	0.0328	0.0342
	FLOW RATE	L/S = 0.0000	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195
	WAVE SPD.	M/S = 0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TIME = 22.561 S.	VELOCITY	M/S = 0.0226	0.0243	0.0261	0.0276	0.0290	0.0302	0.0315	0.0328	0.0342
	FLOW RATE	L/S = 0.0000	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195
	WAVE SPD.	M/S = 0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TIME = 22.840 S.	VELOCITY	M/S = 0.0226	0.0244	0.0261	0.0276	0.0290	0.0302	0.0314	0.0327	0.0341
	FLOW RATE	L/S = 0.0000	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195
	WAVE SPD.	M/S = 0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TIME = 23.118 S.	VELOCITY	M/S = 0.0229	0.0245	0.0262	0.0277	0.0290	0.0301	0.0314	0.0327	0.0341
	FLOW RATE	L/S = 0.0000	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195
	WAVE SPD.	M/S = 0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TIME = 23.397 S.	VELOCITY	M/S = 0.0230	0.0249	0.0263	0.0278	0.0290	0.0301	0.0314	0.0327	0.0341
	FLOW RATE	L/S = 0.0000	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195	0.0195
	WAVE SPD.	M/S = 0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TIME = 23.675 S. DEPTH M = 0.0232 0.0246 0.0263 0.0277 0.0299 0.0301 0.0313 0.0327 0.0341 0.0356 0.0371
 VELOCITY M/S = 0.1444 0.1199 0.1010 0.0905 0.0856 0.0829 0.0807 0.0791 0.0775 0.0760 0.0742
 FLOW RATE L/S = 0.1954 0.1818 0.1669 0.1604 0.1609 0.1639 0.1659 0.1679 0.1704 0.1724 0.1744
 WAVE SPD. M/S = 0.4003 0.4150 0.4289 0.4407 0.4512 0.4613 0.4714 0.4829 0.4945 0.5067 0.5193

TIME = 23.954 S. DEPTH M = 0.0233 0.0249 0.0264 0.0277 0.0299 0.0301 0.0313 0.0327 0.0341 0.0356 0.0371
 VELOCITY M/S = 0.1436 0.1196 0.1020 0.0922 0.0874 0.0845 0.0819 0.0799 0.0779 0.0759 0.0742
 FLOW RATE L/S = 0.1956 0.1823 0.1694 0.1634 0.1642 0.1679 0.1722 0.1759 0.1794 0.1800 0.1811
 WAVE SPD. M/S = 0.4014 0.4159 0.4293 0.4403 0.4510 0.4611 0.4716 0.4828 0.4945 0.5067 0.5192

TIME = 24.232 S. DEPTH M = 0.0234 0.0250 0.0264 0.0277 0.0299 0.0301 0.0313 0.0326 0.0341 0.0356 0.0371
 VELOCITY M/S = 0.1430 0.1195 0.1031 0.0939 0.0891 0.0860 0.0830 0.0797 0.0759 0.0720 0.0682
 FLOW RATE L/S = 0.1999 0.1831 0.1710 0.1665 0.1674 0.1708 0.1745 0.1775 0.1794 0.1805 0.1811
 WAVE SPD. M/S = 0.4025 0.4167 0.4297 0.4408 0.4509 0.4610 0.4715 0.4827 0.4944 0.5067 0.5192

TIME = 24.511 S. DEPTH M = 0.0235 0.0250 0.0265 0.0277 0.0299 0.0300 0.0313 0.0326 0.0341 0.0356 0.0371
 VELOCITY M/S = 0.1423 0.1196 0.1042 0.0957 0.0909 0.0875 0.0841 0.0794 0.0764 0.0722 0.0682
 FLOW RATE L/S = 0.2005 0.1842 0.1733 0.1697 0.1707 0.1737 0.1750 0.1771 0.1804 0.1809 0.1811
 WAVE SPD. M/S = 0.4036 0.4174 0.4300 0.4403 0.4508 0.4609 0.4714 0.4826 0.4944 0.5067 0.5192

TIME = 24.789 S. DEPTH M = 0.0236 0.0251 0.0265 0.0277 0.0299 0.0300 0.0313 0.0326 0.0341 0.0356 0.0372
 VELOCITY M/S = 0.1414 0.1200 0.1055 0.0975 0.0927 0.0890 0.0852 0.0811 0.0768 0.0724 0.0682
 FLOW RATE L/S = 0.2003 0.1855 0.1758 0.1728 0.1739 0.1765 0.1790 0.1807 0.1814 0.1815 0.1812
 WAVE SPD. M/S = 0.4045 0.4181 0.4303 0.4408 0.4507 0.4608 0.4714 0.4826 0.4945 0.5067 0.5193

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11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here) The application of the numerical method of characteristics to the solution of the differential equations defining unsteady flow in partially filled drainage system sized pipes is outlined. The derivation of the flow equations is presented, together with the necessary boundary equation formulation to represent variable inflow, system discharge and leakage flow past a stationary deposited solid. A computer program, written in Fortran, is included, together with typical output, that establishes the applicability of this computational method to unsteady flow analysis in gravity flow drainage systems. Proposals for the extension of the described techniques to the prediction of solid transport and flow attenuation in long pipes are also presented.				
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